Name: ______________________ Date: ____________ Block: _____

Q1Q1: Study Guide — Honors PreCalculus

ALL WORK IS TO BE DONE ON YOUR OWN PAPER

Remember that this is a study GUIDE and not the only material you should study. Studying only the problems that appear on this review guide will not be sufficient. You should also review problems from class starters, notes, and homework assignments for additional practice.

Use the following for 1 – 9 below.

Given the functions $f(x)=3x-5$, $g(x)=\dfrac{2x}{x+1}$, and $h(x)=4x^2-3$, find the following.

1. $f(5x-2)$ $= 3(5x-2)-5$

$15x-6-5$

$\boxed{15x-11}$

2. $\dfrac{f(a+h)-f(a)}{h}$

$f(a+h)=3(a+h)-5$
$3a+3h-5$
$f(a)=3a-5$

$\dfrac{(3a+3h-5)-(3a-5)}{h}$

$\dfrac{\cancel{3a}+3h-\cancel{5}-\cancel{3a}+\cancel{5}}{h}=\dfrac{3h}{h}=\boxed{3}$

3. $\dfrac{g(a+h)-g(a)}{h}$

$g(a+h)=\dfrac{2(a+h)}{(a+h)+1}$
$=\dfrac{2a+2h}{a+h+1}$
$g(a)=\dfrac{2a}{a+1}$

$\dfrac{\dfrac{2a+2h}{a+h+1}-\dfrac{2a}{a+1}}{\dfrac{h}{1}}\cdot\dfrac{(a+h+1)(a+1)}{(a+h+1)(a+1)}$

$\dfrac{(2a+2h)(a+1)-2a(a+h+1)}{h(a+h+1)(a+1)}=\dfrac{\cancel{2a^2}+\cancel{2a}+\cancel{2ah}+2h-\cancel{2a^2}-\cancel{2ah}-\cancel{2a}}{h(a+h+1)(a+1)}$

$\dfrac{2\cancel{h}}{\cancel{h}(a+h+1)(a+1)}$

$=\boxed{\dfrac{2}{(a+h+1)(a+1)}}$

4. $\dfrac{h(a+h)-h(a)}{h}$

$h(a+h)=4(a+h)^2-3$
$4(a^2+2ah+h^2)-3$
$4a^2+8ah+4h^2-3$
$h(a)=4a^2-3$

$\dfrac{\cancel{4a^2}+8ah+4h^2-\cancel{3}-(\cancel{4a^2}-\cancel{3})}{h}$

$=\dfrac{8ah+4h^2}{h}$

$=\boxed{8a+4h}$

5. The average rate of change of *f(x)* over the interval [-2, 6].

$a=-2 \quad b=6$

$f(a)=-11 \quad f(b)=13$

$AROC=\dfrac{13-(-11)}{6-(-2)}=\dfrac{24}{8}=\boxed{3}$

6. The average rate of change of *g(x)* over the interval [3, 8].

$a=3 \quad b=8$

$g(3)=\frac{6}{4}=\frac{3}{2} \quad g(8)=\frac{16}{9}$

$AROC=\dfrac{\frac{16}{9}-\frac{3}{2}}{8-3}=\dfrac{\frac{5}{18}}{\frac{5}{1}}=\dfrac{5}{18}\cdot\dfrac{1}{5}$

$=\boxed{\dfrac{1}{18}}$

7. The average rate of change of $h(x)$ over the interval [-4, 5].

$a=-4 \quad b=5$

$h(-4)=61 \quad h(5)=97$

$$\frac{97-61}{5-(-4)} = \frac{36}{9} = \boxed{4}$$

8. The secant line to the function $g(x)$ that passes through the points (2, g(2)) and (5, g(5)).

$a=2 \quad b=5$

$g(2)=\frac{4}{3} \quad g(5)=\frac{10}{6}=\frac{5}{3}$

$$\text{AROC}=\frac{\frac{5}{3}-\frac{4}{3}}{5-2} = \frac{\frac{1}{3}}{\frac{3}{1}} = \frac{1}{3}\cdot\frac{1}{3} = \frac{1}{9}$$

point: $(2, \frac{4}{3})$

slope = AROC = $\frac{1}{9}$

$$\boxed{y-\frac{4}{3}=\frac{1}{9}(x-2)}$$

9. The secant line to the function $h(x)$ that passes through the points (-1, h(-1)) and (1, h(1)).

$a=-1 \quad b=1$

$h(-1)=1 \quad h(1)=1$

$$\text{AROC}=\frac{1-1}{1-(-1)} = \frac{0}{2} = 0$$

point: $(-1, 1)$

slope = AROC = 0

$y-1=0(x+1)$

$$\boxed{y=1}$$

10. Given $f(x)=4x-1$, find the average rate of change from $x = 1$ to $x = 5$. Then find the average rate of change between $x = a$ and $x = a + h$. Show that the average rate of change between two points on a linear function is the slope of the line.

AROC = 4 , m = 4

$a=1 \quad b=5$

$f(a)=3 \quad f(b)=19$

$$\text{AROC} = \frac{19-3}{5-1} = \frac{16}{4} = \boxed{4}$$

$f(a)=4a-1$

$f(a+h)=4(a+h)-1$

$4a+4h-1$

$$\frac{f(a+h)-f(a)}{h} = \frac{(\cancel{4a}+4h-\cancel{1})-(\cancel{4a}-\cancel{1})}{h}$$

$$= \frac{4h}{h} = \boxed{4}$$

11. Given $g(x)=3-2x^2$, find the average rate of change from $x = 3$ to $x = 3 + h$.

$a=3 \quad b=3+h$

$g(a)=-15 \quad g(b)=3-2(3+h)^2$

$3-2(9+6h+h^2)$

$3-18-12h-2h^2$

$-15-12h-2h^2$

$$\text{AROC}=\frac{(-15-12h-2h^2)-(-15)}{(3+h)-(3)}$$

$$= \frac{-\cancel{15}-12h-2h^2+\cancel{15}}{h} = \frac{-12h-2h^2}{h}$$

$$= \boxed{-12-2h}$$

12. Given $f(x)=\dfrac{1}{x+1}$, find the average rate of change from $x = 4$ to $x = 4 + h$.

$a=4$ $\quad b=4+h$

$f(a)=\frac{1}{5}$ $\quad f(b)=\dfrac{1}{(4+h)+1} = \dfrac{1}{5+h}$

$$AROC = \frac{\frac{1}{5+h}-\frac{1}{5}}{(4+h)-4} = \frac{\frac{1}{5+h}-\frac{1}{5}}{\frac{h}{1}} \cdot \frac{5(5+h)}{5(5+h)}$$

$$\frac{5-(5+h)}{5h(5+h)} = \frac{-h}{5h(5+h)} = \boxed{\frac{-1}{5(5+h)}}$$

13. Find the average rate of change for the function $f(x)=\dfrac{1}{x+1}$ from $x = a$ to $x = a + h$.

$f(a)=\dfrac{1}{a+1}$

$f(a+h)=\dfrac{1}{a+h+1}$

$$AROC = \frac{\frac{1}{a+h+1}-\frac{1}{a+1}}{\frac{h}{1}} \cdot \frac{(a+1)(a+h+1)}{(a+1)(a+h+1)}$$

$$= \frac{a+1-(a+h+1)}{h(a+1)(a+h+1)} = \frac{\cancel{a}+\cancel{1}-\cancel{a}-h-\cancel{1}}{h(a+1)(a+h+1)} = \frac{-h}{h(a+1)(a+h+1)} = \boxed{\frac{-1}{(a+1)(a+h+1)}}$$

14. The table shows the number of CD players sold in a small electronics store from 1989 to 1996.

Year	1989	1990	1991	1992	1993	1994	1995	1996
CD Players Sold	512	520	413	410	468	510	590	607

(a) What was the average rate of change of sales between 1989 and 1996?

$$AROC = \frac{607-512}{1996-1989} = \frac{95}{7} = \boxed{13.57}$$

$a=1989$ $\quad b=1996$

$f(a)=512$ $\quad f(b)=607$

(b) What was the average rate of change of sales between 1989 and 1990?

$$AROC = \frac{520-512}{1990-1989} = \frac{8}{1} = \boxed{8}$$

$a=1989$ $\quad b=1990$

$f(a)=512$ $\quad f(b)=520$

(c) Between which two successive years did CD player sales increase most quickly?

1994 to 1995

(d) Between which two successive years did CD player sales decrease most quickly?

1990 to 1991

Find the intervals over which the following functions are increasing and over which the functions are decreasing. Then identify the critical values.

15. $f(x) = 5x^4 + 8x^3 - 6x^2 - 8x + 2$

Rel. Min. $(-1.424, -1.316)$
Abs. Min. $(0.654, -2.646)$
Rel. Max. $(-0.430, 3.865)$
Inc: $(-1.424, -0.430) \cup (.654, \infty)$
Dec: $(-\infty, -1.424) \cup (-.430, .654)$

16. $f(x) = -3x^3 + 2x^2 + 5x - 1$

Rel Min. $(-0.556, -2.646)$
Rel Max. $(1, 3)$
Inc: $(-0.556, 1)$
Dec: $(-\infty, -0.556) \cup (1, \infty)$

Simplify and identify the domain.

17. $\dfrac{x^2-5x+6}{x^2-4} \cdot \dfrac{x^2+3x+2}{x^2-2x-3}$

$$\frac{\cancel{(x-2)}\cancel{(x-3)}}{\cancel{(x-2)}\cancel{(x+2)}} \cdot \frac{\cancel{(x+1)}\cancel{(x+2)}}{\cancel{(x-3)}\cancel{(x+1)}}$$

$$= \boxed{1, \quad x \neq -1, 3, \pm 2}$$

18. $\dfrac{6x^3-6x^2}{x^4+5x^3} \div \dfrac{3x^2-15x+12}{2x^2+2x-40}$

$3(x^2-5x+4)$
$3(x-4)(x-1)$

$\to 2(x^2+x-20)$
$2(x-4)(x+5)$

$$\frac{6x^2\cancel{(x-1)}}{x^3\cancel{(x+5)}} \cdot \frac{2\cancel{(x-4)}\cancel{(x+5)}}{3\cancel{(x-4)}\cancel{(x-1)}}$$

$$= \frac{12x^2}{3x^3} = \boxed{\frac{4}{x}, \quad x \neq 0, 1, 4, -5}$$

19. $\dfrac{3}{x-4} - \dfrac{4x}{2x+3}$

$$\frac{3(2x+3)}{(x-4)(2x+3)} - \frac{4x(x-4)}{(x-4)(2x+3)}$$

$$\frac{3(2x+3) - 4x(x-4)}{(x-4)(2x+3)}$$

$$\frac{6x+9-4x^2+16x}{(x-4)(2x+3)} = \frac{-4x^2+22x+9}{(x-4)(2x+3)}$$

$$= \boxed{\frac{-(4x^2-22x-9)}{(x-4)(2x+3)}, \quad x \neq 4, \frac{-3}{2}}$$

20. $\dfrac{4}{x-1} + \dfrac{5x}{x+6} - \dfrac{6x^2+7}{x^2+5x-6}$

$(x+6)(x-1)$

$$\frac{4(x+6)}{(x+6)(x-1)} + \frac{5x(x-1)}{(x+6)(x-1)} - \frac{6x^2+7}{(x+6)(x-1)}$$

$$\frac{4(x+6) + 5x(x-1) - (6x^2+7)}{(x+6)(x-1)}$$

$$\frac{4x+24+5x^2-5x-6x^2-7}{(x+6)(x-1)}$$

$$\frac{-x^2-x+17}{(x+6)(x-1)} = \boxed{\frac{-(x^2+x-17)}{(x+6)(x-1)} \quad x \neq -6, 1}$$

CD: $x(x+3)$

21. $\dfrac{\frac{3}{x}+\frac{5x}{x+3}}{\frac{x}{1}} \cdot \dfrac{x(x+3)}{x(x+3)}$

$\dfrac{3(x+3)+5x^2}{x^2(x+3)} = \dfrac{3x+9+5x^2}{x^2(x+3)}$

$\boxed{\dfrac{5x^2+3x+9}{x^2(x+3)},\ x \neq 0,-3}$

CD: $x(x+1)$

22. $\dfrac{\frac{4}{x+1}-\frac{2}{x}}{\frac{5}{1}} \cdot \dfrac{x(x+1)}{x(x+1)}$

$\dfrac{4x-2(x+1)}{5x(x+1)} = \dfrac{4x-2x-2}{5x(x+1)} = \dfrac{2x-2}{5x(x+1)}$

$\boxed{\dfrac{2(x-1)}{5x(x+1)},\ x \neq 0,-1}$

23. $\dfrac{2x(x+2)^4-(x+2)^4}{(2x-1)(x+2)^3} = \dfrac{2x(x+2)-(x+2)}{(2x-1)} = \dfrac{2x^2+4x-x-2}{(2x-1)} = \dfrac{2x^2+3x-2}{2x-1}$

$= \dfrac{(x+2)\cancel{(2x-1)}}{\cancel{2x-1}}$

$\boxed{= x+2,\ x \neq 1/2,-2}$

Solve the following rational equations.

24. $\left[\dfrac{1}{x}+\dfrac{2}{x-1}=3\right] x(x-1)$

$x-1+2x = 3x(x-1)$

$3x-1 = 3x^2-3x$

$0 = 3x^2-6x+1$

$x = \dfrac{-(-6)\pm\sqrt{(-6)^2-4(3)(1)}}{2(3)}$

$x = \dfrac{6\pm\sqrt{24}}{6} = \dfrac{6\pm2\sqrt{6}}{6}$ 4.6

$= \boxed{\dfrac{3\pm\sqrt{6}}{3}}$

25. $\left[\dfrac{x}{x-2}+\dfrac{1}{x+2}=\dfrac{8}{x^2-4}\right](x-2)(x+2)$

$x(x+2)+(x-2)=8$

$x^2+2x+x-2=8$

$x^2+3x-10=0$

$(x+5)(x-2)=0$

$x=\boxed{-5},\ \cancel{2}$

26. $\dfrac{5}{x^2-7x+12}-\dfrac{2}{3-x}=\dfrac{5}{x-4}$

$-x+3$

$-(x-3)$

$\left[\dfrac{5}{(x-3)(x-4)}+\dfrac{2}{x-3}=\dfrac{5}{x-4}\right](x-3)(x-4)$

$5+2(x-4)=5(x-3)$

$5+2x-8=5x-15$

$-3=3x-15$

$12=3x$

$4\neq x$ $\boxed{\text{No Solution}}$