

Name: Answers

Block: \_\_\_\_\_

Date: \_\_\_\_\_

Class Examples: Angle Sum and Difference Formulas

Honors PreCalculus

Find the exact value of each expression.

1.  $\sin 75^\circ = \sin(45^\circ + 30^\circ)$

$$\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

2.  $\cos 105^\circ = \cos(45^\circ + 60^\circ)$

$$\cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ$$

$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

3.  $\tan 195^\circ = \tan(45^\circ + 150^\circ)$

$$\frac{\sin(45^\circ + 150^\circ)}{\cos(45^\circ + 150^\circ)} = \frac{\sin 45^\circ \cos 150^\circ + \cos 45^\circ \sin 150^\circ}{\cos 45^\circ \cos 150^\circ - \sin 45^\circ \sin 150^\circ}$$

$$= \frac{\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)} = \frac{-\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}}{-\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}}$$

$$\frac{-\sqrt{6} + \sqrt{2}}{-\sqrt{6} - \sqrt{2}} \cdot \frac{-\sqrt{6} + \sqrt{2}}{-\sqrt{6} + \sqrt{2}} = \frac{6 - 2\sqrt{12} + 2}{6 - 2}$$

$$= \frac{8 - 4\sqrt{3}}{4} = 2 - \sqrt{3}$$

5.  $\cos \frac{11\pi}{12} = \cos\left(\frac{3\pi}{12} + \frac{8\pi}{12}\right)$

$$= \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right)$$

$$\cos \frac{\pi}{4} \cos \frac{2\pi}{3} - \sin \frac{\pi}{4} \sin \frac{2\pi}{3}$$

$$\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$-\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$-\frac{\sqrt{2} + \sqrt{6}}{4}$$

6.  $\csc\left(-\frac{\pi}{12}\right) = \csc\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right)$

$$= \csc\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$$

$$\sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \sin \frac{\pi}{4} \cos \frac{\pi}{3} - \cos \frac{\pi}{4} \sin \frac{\pi}{3}$$

$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$\text{So, } \csc\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \frac{4}{\sqrt{2} - \sqrt{6}} \cdot \frac{\sqrt{2} + \sqrt{6}}{\sqrt{2} + \sqrt{6}} = \frac{4(\sqrt{2} + \sqrt{6})}{2 - 6}$$

$$= \frac{4(\sqrt{2} + \sqrt{6})}{-4} = -(\sqrt{2} + \sqrt{6})$$

$$7. \sin \frac{19\pi}{12} = \sin \left( \frac{3\pi}{12} + \frac{16\pi}{12} \right)$$

$$= \sin \left( \frac{\pi}{4} + \frac{4\pi}{3} \right)$$

$$= \sin \frac{\pi}{4} \cos \frac{4\pi}{3} + \cos \frac{\pi}{4} \sin \frac{4\pi}{3}$$

$$\left( \frac{\sqrt{2}}{2} \right) \left( -\frac{1}{2} \right) + \left( \frac{\sqrt{2}}{2} \right) \left( -\frac{\sqrt{3}}{2} \right)$$

$$\frac{-\sqrt{2}}{4} + \frac{-\sqrt{6}}{4}$$

$$\frac{-\sqrt{2} - \sqrt{6}}{4}$$

$$8. \cos \left( -\frac{5\pi}{12} \right) = \cos \left( \frac{3\pi}{12} - \frac{8\pi}{12} \right)$$

$$= \cos \left( \frac{\pi}{4} - \frac{2\pi}{3} \right)$$

$$\cos \frac{\pi}{4} \cos \frac{2\pi}{3} + \sin \frac{\pi}{4} \sin \frac{2\pi}{3}$$

$$\left( \frac{\sqrt{2}}{2} \right) \left( -\frac{1}{2} \right) + \left( \frac{\sqrt{2}}{2} \right) \left( \frac{\sqrt{3}}{2} \right)$$

$$\frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$\frac{-\sqrt{2} + \sqrt{6}}{4}$$

$$9. \sin \frac{\pi}{18} \cos \frac{5\pi}{18} + \cos \frac{\pi}{18} \sin \frac{5\pi}{18}$$

$$\sin \left( \frac{\pi}{18} + \frac{5\pi}{18} \right)$$

$$\sin \frac{6\pi}{18}$$

$$\sin \frac{\pi}{3}$$

$\frac{\pi}{2}$  (0,1) Verify the following trigonometric identities.

$$10. \tan \left( \frac{\pi}{2} - u \right) = \cot u$$

$$\frac{\sin \left( \frac{\pi}{2} - u \right)}{\cos \left( \frac{\pi}{2} - u \right)} = \frac{\sin \frac{\pi}{2} \cos u - \cos \frac{\pi}{2} \sin u}{\cos \frac{\pi}{2} \cos u + \sin \frac{\pi}{2} \sin u}$$

$$\frac{(1)\cos u - (0)\sin u}{(0)\cos u + (1)\sin u} = \frac{\cos u}{\sin u}$$

$$= \cot u \checkmark$$

$$11. \sin(x+y) - \sin(x-y) = 2 \cos x \sin y$$

$$\sin x \cos y + \cos x \sin y - (\sin x \cos y - \cos x \sin y)$$

$$\sin x \cos y + \cos x \sin y - \sin x \cos y + \cos x \sin y$$

$$2 \cos x \sin y \checkmark$$

$$12. \cos\left(x + \frac{\pi}{6}\right) + \sin\left(x - \frac{\pi}{3}\right) = 0$$

$$\begin{aligned} & \cos x \cos \frac{\pi}{6} - \sin x \sin \frac{\pi}{6} + \sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3} \\ & \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x + \frac{1}{2} \sin x - \frac{\sqrt{3}}{2} \cos x \end{aligned}$$

$$= 0 \checkmark$$

$$13. \tan\left(x - \frac{\pi}{4}\right) = \frac{\tan x - 1}{\tan x + 1}$$

$$\frac{\sin\left(x - \frac{\pi}{4}\right)}{\cos\left(x - \frac{\pi}{4}\right)}$$

$$= \frac{\sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4}}{\cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4}}$$

$$= \frac{\frac{\sqrt{2}}{2} \sin x - \frac{\sqrt{2}}{2} \cos x}{\frac{\sqrt{2}}{2} \cos x + \frac{\sqrt{2}}{2} \sin x}$$

$$\frac{\sqrt{2} \sin x - \sqrt{2} \cos x}{\sqrt{2} \cos x + \sqrt{2} \sin x}$$

$$\frac{\sqrt{2} (\sin x - \cos x)}{\sqrt{2} (\cos x + \sin x)} = \frac{\sin x - \cos x}{\cos x + \sin x}$$

$$= \frac{\sin x - \cos x}{\cos x + \sin x} \cdot \frac{\sec x}{\sec x}$$

$$\frac{\frac{\sin x}{\cos x} - 1}{1 + \frac{\sin x}{\cos x}} = \frac{\tan x - 1}{1 + \tan x}$$

$$= \frac{\tan x - 1}{\tan x + 1} \checkmark$$

