Name: ______________________ Date: ____________ Block: ____

Class Examples: Compound Interest — **Honors PreCalculus**

One of the most important applications of exponential functions is "compound interest." Compound interest is the method by which banks and financial institutions determine how much interest to pay you for money in an account or how much money you owe when taking out a loan. Therefore, it is very important for you to know how compound interest works!

Assume you invest a certain amount of money in an interest bearing account. The money you invest is known as the principle. At regular intervals, the bank/financial institution will take a certain percentage (known as the interest rate) of the amount in the account and add that back to the money in your account. This percentage added back is like free money! You did nothing to earn it... other than leave your money in your account. Each time this percentage is taken and added back, you increase the amount of money in your account (and keep in mind that it is taking a percentage of what is currently in the account--not just of the original amount--so it is more each time). And such is the power of compound interest! (Though it can work against you because loans where you owe money can work the same way.)

The formula for compound interest is.... $A = P\left(1+\frac{r}{n}\right)^{nt}$

In this formula, A represents the current amount in the account, P represents the principle, r represents the interest rate, n represents the number of times compounded per year, and t represents the time in years. Interest can be compounded annually (n = 1), semiannually (n = 2), quarterly (n = 4), monthly (n = 12), weekly (n = 52), daily (n = 365), etc.

It is also possible for interest to be compounded continuously. When this is the case, we use the formula... $A = Pe^{rt}$*. For this equation, A still represents the amount in the account and P sill refers to the principle. The variable r is still the interest rate and t is still time in years. The "e" in the equation is the number that is associated with the natural logarithm function. You will notice that anytime you see something continually increasing or decreasing, we will use the number e as part of the function.*

1. You invest $4000 in an account with an interest rate of 4%, compounded semiannually. How much money is in the account at the end of 20 years?

$P = 4000$
$r = .04$
$n = 2$

$A = 4000\left(1+\frac{.04}{2}\right)^{2t}$
$A = 4000(1.02)^{2t}$

$t = 20$
$A = 4000(1.02)^{2(20)}$
$A = 8832.16$

$8832.16 is in the account after 20 years.

2. When you decided to invest your money in #1, there was also the option of investing in an account with an interest rate of 3.75%, compounded continuously. How much more or how much less would there be in the account at the end of 20 years?

$P = 4000$
$r = .0375$

$A = 4000e^{.0375t}$

$t = 20$
$A = 4000e^{.0375(20)}$
$A = 8468.00$

$8468.00 is in the account after 20 years, $364.16 less than the option in #1.

3. Given that an account has an interest rate of 5.25%, if you invest $10,000 in the account, how much money will be in the account after 15 years if the account is compounded monthly? Continuously?

$P = 10000$
$r = .0525$
$n = 12$

$A = 10000\left(1+\frac{.0525}{12}\right)^{12t}$
$A = 10000(1.004375)^{12t}$
$t = 15$ $A = 10000(1.004375)^{12(15)}$
$\boxed{A = \$21941.23 \text{ (monthly)}}$

$P = 10000$
$r = .0525$

$A = 10000e^{.0525t}$
$t = 15$ $A = 10000e^{.0525(15)}$
$\boxed{A = 21978.95 \text{ (continuously)}}$

4. You invest \$5000 in an account that has an interest rate of 2.45%, compounded annually. When will the amount in the account grow to be \$8000?

$P=5000$
$r=.0245$
$n=1$

$A=5000\left(1+\frac{.0245}{1}\right)^t$

$A=5000(1.0245)^t$

$A=8000 \rightarrow 8000=5000(1.0245)^t$

$1.6=1.0245^t$

$\log_{1.0245}1.6=t$

$19.42 \text{ years}=t$

5. Suppose you invest \$15,000 in an account that has an interest rate of 6.25%, compounded continuously. How long will it take for you to double your investment?

$P=15000$
$r=.0625$

$A=15000e^{.0625t}$

double $\rightarrow$ \$30000 = A

$30000=15000e^{.0625t}$

$2=e^{.0625t}$

$\ln 2=.0625t$

$.693=.0625t$

$11.09 \text{ years}=t$

6. How long would it take for you to triple a \$3,500 investment if you were to invest it in an account that has an interest rate of 3.65%, compounded quarterly?

$P=3500$
$r=.0365$
$n=4$

$A=3500\left(1+\frac{.0365}{4}\right)^{4t}$

$A=3500(1.009125)^{4t}$

triple $\rightarrow$ $3(3500)=\$10500=A$

$10500=3500(1.009125)^{4t}$

$3=1.009125^{4t}$

$\log_{1.009125}3=4t$

$120.94=4t$

$30.236 \text{ years}=t$

7. How long would it take to double an investment in an account with an interest rate of 7%, compounded continuously?

$r=.07$

$\frac{2P}{P}=\frac{Pe^{.07t}}{P}$

$2=e^{.07t}$

$\ln 2=.07t$

$.693=.07t$

$9.9 \text{ years}=t$

8. The resident population of the United States in 2010 was 309 million people and was growing at a rate of 0.9% per year. Assuming that this growth rate continues, the model $P(t)=309(1.009)^{t-2010}$ represents the population P (in millions of people) in years t.

a) According to this model, when will the population of the United States be 419 million?

$419=309(1.009)^{t-2010}$

$1.356=1.009^{t-2010}$

$\log_{1.009}1.356=t-2010$

$33.99=t-2010$

$2043.99=t$

b) According to this model, when will the population of the United States be 488 million?

$488=309(1.009)^{t-2010}$

$1.579=1.009^{t-2010}$

$\log_{1.009}1.579=t-2010$

$51.003=t-2010$

$2061.003=t$

c) What will be the population in the year 2030?

$P(2030)=309(1.009)^{2030-2010}$

$=309(1.009)^{20}$

$=369.64$

In 2030 the population is expected to be 369.64 million people.