Name: Answers

Block: ____

Date: ______________

Class Examples: Inverse Functions

Honors PreCalculus

Use the horizontal line test to state whether the inverse of each is a function.

1. $f(x) = -x - 2$

yes

2. $f(x) = -x^2 + 3,\ x \geq 0$

yes

3. $f(x) = |x| + 1$

no

Verify that f and g are inverse functions by showing that $f(g(x)) = g(f(x)) = x$

4. $f(x) = 2x - 4;\ \ g(x) = \frac{1}{2}x + 2$

$f(g(x)) = 2(\frac{1}{2}x+2) - 4$
$x + 4 - 4$
$= x$ ✓

$g(f(x)) = \frac{1}{2}(2x-4) + 2$
$x - 2 + 2$
$= x$ ✓

5. $f(x) = \frac{x-5}{2x+3},\ \ g(x) = \frac{3x+5}{1-2x}$

$f(g(x)) = \dfrac{\frac{3x+5}{1-2x} - 5}{2\left(\frac{3x+5}{1-2x}\right) + 3} = \dfrac{\frac{3x+5}{1-2x} - \frac{5}{1}}{\frac{6x+10}{1-2x} + 3}$

$\dfrac{3x+5-5(1-2x)}{6x+10+3(1-2x)} = \dfrac{3x+5-5+10x}{6x+10+3-6x} = \dfrac{13x}{13} = x$ ✓

$g(f(x)) = \dfrac{3\left(\frac{x-5}{2x+3}\right)+5}{1-2\left(\frac{x-5}{2x+3}\right)} = \dfrac{\frac{3x-15}{2x+3} + 5}{1 - \frac{2x-10}{2x+3}}$

$\dfrac{3x-15+5(2x+3)}{2x+3-(2x-10)} = \dfrac{3x-15+10x+1}{2x+3-2x+10}$

$= \dfrac{13x}{13} = x$ ✓

6. $f(x) = x^2 + 5;\ \ g(x) = \sqrt{x-5}$

$f(g(x)) = (\sqrt{x-5})^2 + 5$
$= x - 5 + 5$
$= x$ ✓

$g(f(x)) = \sqrt{(x^2+5) - 5}$
$= \sqrt{x^2}$
$= x$ ✓

Find the inverse of the following functions. Tell whether or not the inverse is a function. State the domain of f and f^{-1}

7. $y = x^2 + 9; x \geq 0$ D: $x \geq 0$

$x = y^2 + 9$
$x - 9 = y^2$
$\sqrt{x-9} = y$
$f^{-1}(x) = \sqrt{x-9}$

D: $x - 9 \geq 0$
$x \geq 9$

8. $y = \frac{4}{x+2}$

D: $\{x \mid x \neq -2\}$

$y+2 \cdot x = \frac{4}{y+2} \cdot y+2$

$\frac{x(y+2)}{x} = \frac{4}{x}$

$y + 2 = \frac{4}{x}$

$y = \frac{4}{x} - 2$

$f^{-1}(x) = \frac{4}{x} - 2$

D: $\{x \mid x \neq 0\}$

9. $y = \frac{-3x-4}{x-2}$

$y-2 \cdot x = \frac{-3y-4}{y-2} \cdot y-2$

$x(y-2) = -3y-4$

$xy - 2x = -3y-4$

$xy + 3y = 2x-4$

$y(x+3) = 2x-4$

$y = \frac{2x-4}{x+3}$

$f^{-1}(x) = \frac{2x-4}{x+3}$

10. $y = \sqrt{x-3}$

$x = \sqrt{y-3}$

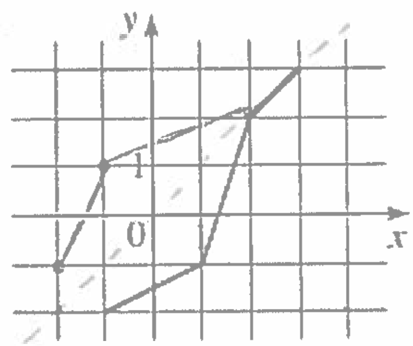
$x^2 = y-3$

$x^2 + 3 = y$

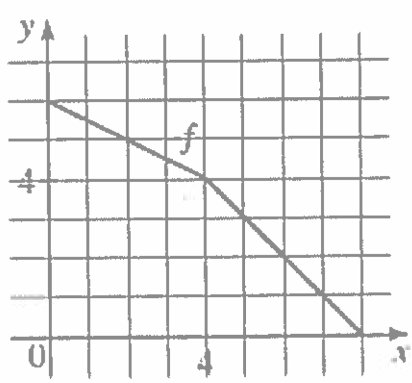
$f^{-1}(x) = x^2 + 3$

11. The graph of a one-to-one function is given. Draw the graph of its inverse.

(reflect over line $y=x$)



12. Use the graph below to find the following values.



(a) $f^{-1}(2)$

6

(b) $f^{-1}(5)$

2

(c) $f^{-1}(6)$

0

13. Use the table at the right to find the following values.

x	1	2	3	4	5	6
$f(x)$	4	6	2	5	0	1

a) $f^{-1}(5) = 4$

b) $f^{-1}(0) = 5$

c) $f^{-1}(f(1)) = f^{-1}(4) = 1$

d) $f(f^{-1}(6)) = f(2) = 6$

e) $f^{-1}(f^{-1}(1)) = f^{-1}(6) = 2$

f) $f^{-1}(f^{-1}(0)) = f^{-1}(5) = 4$

Partner Activity:

14. Describe and correct the error. Given $f(x) = \sqrt{x-6}$, then $f^{-1}(x) = \frac{1}{\sqrt{x-6}}$

$x = \sqrt{y-6}$

$x^2 = y-6$

$x^2 + 6 = y$

$f^{-1}(x) = x^2 + 6$

15. Is the inverse of a function a function?

Not if the original function is one-to-one.

16. Do all functions have inverses?

Yes but the inverse may not be a function

17. Do all linear functions have inverses?

Yes, though horizontal lines would have an inverse that is not a function

18. What is the slope of the inverse of a linear function? → the slope is the reciprocal of the original slope

$y = mx + b$

$x = my + b$

$x - b = my$

$y = \frac{x-b}{m} = \frac{x}{m} - \frac{b}{m}$

$y = \frac{1}{m}x - \frac{b}{m}$