

## Class Examples: Properties of Logarithms

## Honors PreCalculus

Use the Properties of Logarithms to evaluate each expression.

1.  $\log_4 192 - \log_4 3$

$$\log_4 \frac{192}{3}$$

$$\log_4 64 = x$$

$$4^x = 64$$

$$x = 3$$

2.  $\log_3 \sqrt{27}$

$$\log_3 27^{1/2}$$

$$\frac{1}{2} \log_3 27$$

$$3^x = 27$$

$$x = 3$$

$$\frac{1}{2}(3) = \frac{3}{2}$$

3.  $\log_2 6 - \log_2 15 + \log_2 20$

$$\log_2 \frac{6 \cdot 20}{15}$$

$$\log_2 8 = x$$

$$2^x = 8$$

$$x = 3$$

4.  $\log \frac{1}{\sqrt{1000}}$

$$\log 1000^{-1/2}$$

$$-1/2 \log 1000$$

$$10^x = 1000$$

$$x = 3$$

$$-1/2(3) = -3/2$$

Expand each logarithm.

5.  $\log_9 \frac{1}{3} + 3 \log_9 3$

$$\log_9 \frac{1}{3}(3^3)$$

$$\log_9 \frac{1}{3}(27)$$

$$\log_9 9 = x$$

$$9^x = 9$$

$$x = 1$$

6.  $\log_3 100 - \log_3 18 - \log_3 50$

$$\log_3 \frac{100}{18 \cdot 50}$$

$$\log_3 \frac{1}{9} = x$$

$$3^x = \frac{1}{9}$$

$$x = -2$$

7.  $\log_2 (xy)^{10}$

$$10 \log_2 x + 10 \log_2 y$$

8.  $\ln \sqrt[3]{3r^2s}$

$$\frac{1}{3} (\ln 3 + 2 \ln r + \ln s)$$

$$\frac{1}{3} \ln 3 + \frac{2}{3} \ln r + \frac{1}{3} \ln s$$

9.  $\log_3 \frac{5x^3 \sqrt{y}}{9z^4}$

$$\log_3 5 + 3 \log_3 x + \frac{1}{2} \log_3 y - \log_3 9 - 4 \log_3 z$$

$$\frac{3^x = 9}{x = 2}$$

$$\log_3 5 + 3 \log_3 x + \frac{1}{2} \log_3 y - 2 - 4 \log_3 z$$

10.  $\log \left( \frac{a^2}{b^4 \sqrt{c}} \right)$

$$2 \log a - 4 \log b - \frac{1}{2} \log c$$

11.  $\ln \left( \frac{x^3 \sqrt{x-1}}{3x+4} \right)$

$$3 \ln x + \frac{1}{2} \ln (x-1) - \ln (3x+4)$$

12.  $\log \left( \frac{10^x}{x(x^2+1)(x^4+2)} \right)$

$$x \log 10 - \log x - \log (x^2+1) - \log (x^4+2)$$

$$\frac{10^x = 10}{x = 1}$$

$$x - \log x - \log (x^2+1) - \log (x^4+2)$$

Condense: Write each logarithm as a single logarithm.

13.  $2(\log_5 x + 2 \log_5 y - 3 \log_5 z)$

$$2 \log_5 x + 4 \log_5 y - 6 \log_5 z$$

$$\log_5 \frac{x^2 y^4}{z^6}$$

14.  $2 \log a - 4 \log b - \frac{1}{2} \log c$

$$\log \frac{a^2}{b^4 \sqrt{c}}$$

15.  $2[\ln x + \ln(x-2)] - \frac{1}{2} \ln(2x-3)$

$$2 \ln x + 2 \ln(x-2) - \frac{1}{2} \ln(2x-3)$$

$$\ln \frac{x^2 (x-2)^2}{\sqrt{2x-3}}$$

16.  $4 \log x - \frac{1}{3} \log(x^2 + 1) + 2 \log(x-1)$

$$\log \frac{x^4 (x-1)^2}{\sqrt[3]{x^2+1}}$$

17.  $4 \log_2 x - \frac{1}{3} [\log_2(2x+1) + \log_2 x] + \frac{1}{2} \log_2(x+3)$

$$4 \log_2 x - \frac{1}{3} \log_2(2x+1) - \frac{1}{3} \log_2 x + \frac{1}{2} \log_2(x+3)$$

$$\log_2 \frac{x^4 \sqrt{x+3}}{\sqrt[3]{x(2x+1)}}$$

Write each in simplest form.

18.  $21 \log_3 \sqrt[3]{x} + \log_3(9x^2) - \log_3 9$

$$\log_3 \frac{x^{21/3} \cdot 9x^2}{9} \rightarrow \log_3 x^7 \cdot x^2$$

$$\rightarrow \log_3 x^9$$

$$\rightarrow \boxed{9 \log_3 x}$$

19.  $\log(\sqrt{x} - \sqrt{x-1}) + \log(\sqrt{x} + \sqrt{x-1})$

$$\log[(\sqrt{x} - \sqrt{x-1})(\sqrt{x} + \sqrt{x-1})]$$

$$\log[x - (x-1)]$$

$$\log 1$$

$$\boxed{0}$$

20.  $\log_5(x^2 - 1) - \log_5(x - 1)$

$$\log_5 \frac{x^2 - 1}{x - 1}$$

$$\log_5 \frac{(x-1)(x+1)}{x-1}$$

$$\boxed{\log_5(x+1)}$$

**Partner Activity:**

1. Without using a calculator, find the exact value of  $\frac{\log_3 81 - \log_{\pi} 1}{\log_{2\sqrt{2}} 8 - \log 0.001} = \boxed{\frac{4}{5}}$

$$3^x = 81 \quad \pi^x = 1$$
$$x = 4 \quad x = 0$$

$$(2\sqrt{2})^x = 8 \quad 10^x = \frac{1}{1000}$$
$$x = 2 \quad x = -3$$

2. Without using a calculator, determine which is the greater number:  $\log_4 60$  or  $\log_3 40$

$$4^x = 60 \quad 3^x = 40$$
$$4^3 = 64 \quad 3^3 = 27$$

close to 3       $3^4 = 81$

↑  
larger

