Name: Answers Block: ____

Date: ____

Class Examples: Solving Trigonometric Equations (Day 2) **Honors PreCalculus**

Solve each equation.

1. $\tan\frac{\theta}{2} = \sqrt{3}$

$\tan\theta = \sqrt{3}$

S | A $\frac{\pi}{3}$

T | C

$\frac{4\pi}{3}$

$\frac{\theta}{2} = \frac{\pi}{3} \pm 2\pi n \quad \frac{\theta}{2} = \frac{4\pi}{3} \pm 2\pi n$

$\boxed{\theta = \frac{2\pi}{3} \pm 4\pi n \quad \theta = \frac{8\pi}{3} \pm 4\pi n \quad \text{or} \quad \theta = \frac{2\pi}{3} \pm 2\pi n}$

2. $4\sin\theta + 3\sqrt{3} = \sqrt{3}$

$4\sin\theta = -2\sqrt{3}$

$\sin\theta = -\frac{\sqrt{3}}{2}$

S | A $\frac{\pi}{3}$

$\frac{4\pi}{3}$ $-\frac{\pi}{3}$

$\boxed{\theta = \frac{4\pi}{3} \pm 2\pi n, \quad -\frac{\pi}{3} \pm 2\pi n}$

3. $\sin^2\theta - 1 = 0$

$\sin^2\theta = 1$

$\sin\theta = \pm 1$

(0,1)

(0,-1)

$\boxed{\theta = \frac{\pi}{2} \pm 2\pi n, \quad \frac{3\pi}{2} \pm 2\pi n \quad \text{or} \quad \theta = \frac{\pi}{2} \pm \pi n}$

4. $(\cos\theta + 1)\left(\csc\theta - \frac{1}{2}\right) = 0$

$\cos\theta = -1 \qquad \csc\theta = \frac{1}{2}$

(-1,0)

$\boxed{\theta = \pi \pm 2\pi n} \qquad \sin\theta = 2$

undefined

5. $\sin^2\theta = 2\cos\theta + 2$

$(1 - \cos^2\theta) = 2\cos\theta + 2$

$0 = \cos^2\theta + 2\cos\theta + 1$

$(\cos\theta + 1)(\cos\theta + 1) = 0$

$\cos\theta = -1 \qquad \cos\theta = -1$

(-1,0)

$\boxed{\theta = \pi \pm 2\pi n}$

6. $2\cos^2\theta - 7\cos\theta - 4 = 0$

$-8 \times 1 = -8$

$-8 + 1 = -7$

$\frac{2\cos\theta}{-8} = \frac{\cos\theta}{-4}$

$\frac{2\cos\theta}{1} =$

$(\cos\theta - 4)(2\cos\theta + 1) = 0$

$\cos\theta - 4 = 0 \qquad 2\cos\theta + 1 = 0$

$\cos\theta = 4 \qquad \cos\theta = -\frac{1}{2}$

undefined

$\frac{2\pi}{3}$ S | A $\frac{\pi}{3}$

T | C

$\frac{4\pi}{3}$

$\boxed{\theta = \frac{2\pi}{3} \pm 2\pi n, \quad \frac{4\pi}{3} \pm 2\pi n}$

7. $2\cos(3\theta) + 1 = 0$

$\frac{2\pi}{3}$ S | A $\frac{\pi}{3}$
T | C
$\frac{4\pi}{3}$

$\cos(3\theta) = -\frac{1}{2}$

$3\theta = \frac{2\pi}{3} \pm 2\pi n \qquad 3\theta = \frac{4\pi}{3} \pm 2\pi n$

$\boxed{\theta = \frac{2\pi}{9} \pm \frac{2\pi n}{3} \qquad \frac{4\pi}{9} \pm \frac{2\pi n}{3}}$

8. In 1983, George Ferris engineered the Ferris Wheel. It was 250 feet in diameter. If the wheel makes 1 revolution every 40 seconds, then the function $h(t) = 125\sin\left(0.157t - \frac{\pi}{2}\right) + 125$ represents the height h, in feet, of a seat on the wheel as a function of time t, where t is measured in seconds. The ride begins at $t = 0$. X-min = 0 X-max=40

radians

a) During the first 40 seconds of the ride, at what time t is an individual on the Ferris Wheel exactly 125 feet above the ground?

π (-1,0) 0 (1,0)

$125 = 125\sin(.157t - \frac{\pi}{2}) + 125$

$0 = \sin(.157t - \frac{\pi}{2})$

$0 = .157t - \frac{\pi}{2} \qquad \pi = .157t - \frac{\pi}{2}$

$\frac{\pi}{2} = .157t \qquad \frac{3\pi}{2} = .157t$

$\boxed{10.005s = t \quad \text{or} \quad 30.015s = t}$

X-min = 0 X-max = 80

b) During the first 80 seconds of the ride, at what time t is an individual on the Ferris Wheel exactly 250 feet above the ground? coterminal

(0,1)

$250 = 125\sin(.157t - \frac{\pi}{2}) + 125$

$1 = \sin(.157t - \frac{\pi}{2})$

$\frac{\pi}{2} = .157t - \frac{\pi}{2} \qquad \frac{5\pi}{2} = .157t - \frac{\pi}{2}$

$\pi = .157t \qquad 3\pi = .157t$

$\boxed{20.010s = t \qquad 60.030s = t}$

* 9. The tide, or depth of the ocean near the shore, changes throughout the day. The depth of the Bay of Fundy can be modeled by $d = 35 - 28\cos\frac{\pi}{6}t$ where d is the water depth in feet and t is the time in hours. Consider a day in which $t = 0$ represents 12:00 A.M.

a) For that day, when do the high and low tides occur?

X-min: 0 y-min: 0
X-max: 24 y-max: 65
Scale: 1 Scale: 5

Max: 6, 18 Min: 12

$\boxed{\text{High Tide: 6:00am, 6:00pm} \quad \text{Low Tide: 12:00 noon}}$

b) At what time(s) is the water depth 3.5 feet?

$3.5 = 35 - 28\cos\frac{\pi}{6}t$

$1.125 = \cos\frac{\pi}{6}t$

$\boxed{\text{undefined}}$

10. The number of hours of daylight in Boston during one year is given by $y = 3sin\left[\frac{2\pi}{365}(x-79)\right]+12$ where x is the number of days after January 1.

a) Within a year, when does Boston have 10.5 hours of daylight? Give your answer in days after January 1 and round to the nearest day.

$x_{min}=0$ $y_{min}=-2$

$x_{max}=365$ $y_{max}=12$

$10.5 = 3\sin\left[\frac{2\pi}{365}(x-79)\right]+12$

$-0.5 = \sin\left[\frac{2\pi}{365}(x-79)\right]$

S | A $\frac{\pi}{6}$

T | C

$\frac{7\pi}{6}$ $\frac{11\pi}{6}$ / $-\frac{\pi}{6}$

$\frac{7\pi}{6} = \frac{2\pi}{365}(x-79)$ $-\frac{\pi}{6} = \frac{2\pi}{365}(x-79)$

$\frac{2555}{12} = x-79$ $-\frac{365}{12} = x-79$

$291.917 = x$ $48.583 = x$

$\boxed{x = 48.58 \text{ days after Jan 1} \quad x = 291.9 \text{ days after Jan 1}}$

b) Within a year, when does Boston have 13.5 hours of daylight? Give your answer in days after January 1 and round to the nearest day.

$13.5 = 3\sin\left[\frac{2\pi}{365}(x-79)\right]+12$

$\frac{1}{2} = \sin\left[\frac{2\pi}{365}(x-79)\right]$

$\frac{\pi}{6} = \frac{2\pi}{365}(x-79)$ $\frac{5\pi}{6} = \frac{2\pi}{365}(x-79)$

$\frac{365}{12} = x-79$ $\frac{1825}{12} = x-79$

$\boxed{x = 109.417 \text{ days after Jan 1st} \quad x = 231.083 \text{ days after Jan 1st}}$

11. A ball on a spring is pulled 4 inches below its rest position and then released. After t seconds, the ball's distance, d, in inches from its resting position is given by $d = -4\cos\left(\frac{\pi}{3}t\right)$

a) Find all values of t for which the ball is 2 inches above its rest position.

$2 = -4\cos\left(\frac{\pi}{3}t\right)$

$-\frac{1}{2} = \cos\left(\frac{\pi}{3}t\right)$

$\frac{2\pi}{3}$ S | A $\frac{\pi}{3}$

$\frac{4\pi}{3}$ T | C

$\frac{2\pi}{3} = \frac{\pi}{3}t$ $\frac{4\pi}{3} = \frac{\pi}{3}t$

$2 = t$ $4 = t$

$\boxed{2 \text{ seconds} \pm 2\pi n \quad 4 \text{ seconds} \pm 2\pi n}$

b) Find all values of t for which the ball is 2 inches below its rest position.

$-2 = -4\cos\left(\frac{\pi}{3}t\right)$

$\frac{1}{2} = \cos\left(\frac{\pi}{3}t\right)$

S | A $\frac{\pi}{3}$

T | C $\frac{5\pi}{3}$

$\frac{\pi}{3} = \frac{\pi}{3}t$ $\frac{5\pi}{3} = \frac{\pi}{3}t$

$1 = t$ $5 = t$

$\boxed{1 \text{ second} \pm 2\pi n \quad 5 \text{ seconds} \pm 2\pi n}$