

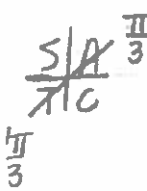
Solve each equation.

1. $\tan \frac{\theta}{2} = \sqrt{3}$

$\tan \theta = \sqrt{3}$

$\frac{\theta}{2} = \frac{\pi}{3} \pm 2\pi n \quad \frac{\theta}{2} = \frac{4\pi}{3} \pm 2\pi n$

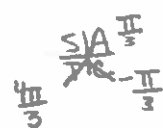
$\theta = \frac{2\pi}{3} \pm 4\pi n \quad \theta = \frac{8\pi}{3} \pm 4\pi n$
or
 $\theta = \frac{2\pi}{3} \pm 2\pi n$



2. $4 \sin \theta + 3\sqrt{3} = \sqrt{3}$

$4 \sin \theta = -2\sqrt{3}$

$\sin \theta = -\frac{\sqrt{3}}{2}$



$\theta = \frac{4\pi}{3} \pm 2\pi n$
 $-\frac{2\pi}{3} \pm 2\pi n$

3. $\sin^2 \theta - 1 = 0$

$\sin^2 \theta = 1$

$\sin \theta = \pm 1$

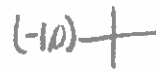
$\theta = \frac{\pi}{2} \pm 2\pi n$
 $\frac{3\pi}{2} \pm 2\pi n$
or
 $\theta = \frac{\pi}{2} \pm \pi n$



4. $(\cos \theta + 1)(\csc \theta - \frac{1}{2}) = 0$

$\cos \theta = -1$

$\csc \theta = \frac{1}{2}$



$\theta = \pi \pm 2\pi n$

$\sin \theta = 2$

↑
undefined

5. $\sin^2 \theta = 2 \cos \theta + 2$

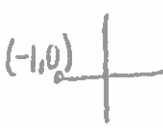
$(1 - \cos^2 \theta) = 2 \cos \theta + 2$

$0 = \cos^2 \theta + 2 \cos \theta + 1$

$(\cos \theta + 1)(\cos \theta + 1) = 0$

$\cos \theta = -1 \quad \cos \theta = -1$

$\theta = \pi \pm 2\pi n$



6. $2 \cos^2 \theta - 7 \cos \theta - 4 = 0$

$-\frac{8}{-8} \times \frac{1}{-1} = -8$

$\frac{2 \cos \theta}{-8} = \frac{\cos \theta}{-4}$

$-\frac{8}{-8} + \frac{1}{-1} = -7$

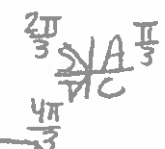
$(\cos \theta - 4)(2 \cos \theta + 1) = 0$

$\cos \theta - 4 = 0 \quad 2 \cos \theta + 1 = 0$

$\cos \theta = 4 \quad \cos \theta = -\frac{1}{2}$

undefined

$\theta = \frac{2\pi}{3} \pm 2\pi n$
 $\frac{4\pi}{3} \pm 2\pi n$



7. $2 \cos(3\theta) + 1 = 0$

$\cos(3\theta) = -\frac{1}{2}$

$3\theta = \frac{2\pi}{3} \pm 2\pi n$ $3\theta = \frac{4\pi}{3} \pm 2\pi n$

$\theta = \frac{2\pi}{9} \pm \frac{2\pi n}{3}$ $\frac{4\pi}{9} \pm \frac{2\pi n}{3}$

$\frac{2\pi}{3}$ $\frac{\pi}{3}$
 $\frac{4\pi}{3}$
 $\frac{S/A}{\pi/C}$

8. In 1983, George Ferris engineered the Ferris Wheel. It was 250 feet in diameter. If the wheel makes 1 revolution every 40 seconds, then the function $h(t) = 125 \sin\left(0.157t - \frac{\pi}{2}\right) + 125$ represents the height h , in feet, of a seat on the wheel as a function of time t , where t is measured in seconds. The ride begins at $t = 0$. $x\text{-min} = 0$ $x\text{-max} = 40$

radians

a) During the first 40 seconds of the ride, at what time t is an individual on the Ferris Wheel exactly 125 feet above the ground?

$125 = 125 \sin\left(0.157t - \frac{\pi}{2}\right) + 125$

$0 = \sin\left(0.157t - \frac{\pi}{2}\right)$

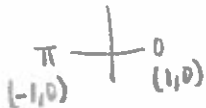
$0 = 0.157t - \frac{\pi}{2}$

$\pi = 0.157t - \frac{\pi}{2}$

$\frac{\pi}{2} = 0.157t$

$\frac{3\pi}{2} = 0.157t$

$10.005s = t$ or $30.015s = t$



b) During the first 80 seconds of the ride, at what time t is an individual on the Ferris Wheel exactly 250 feet above the ground? $x\text{-min} = 0$ $x\text{-max} = 80$ *coterminal*

$250 = 125 \sin\left(0.157t - \frac{\pi}{2}\right) + 125$

$1 = \sin\left(0.157t - \frac{\pi}{2}\right)$

$\frac{\pi}{2} = 0.157t - \frac{\pi}{2}$

$\frac{5\pi}{2} = 0.157t - \frac{\pi}{2}$

$\pi = 0.157t$

$3\pi = 0.157t$

$20.010s = t$

$60.030s = t$



* 9. The tide, or depth of the ocean near the shore, changes throughout the day. The depth of the Bay of Fundy can be modeled by $d = 35 - 28 \cos \frac{\pi}{6} t$ where d is the water depth in feet and t is the time in hours. Consider a day in which $t = 0$ represents 12:00 A.M.

a) For that day, when do the high and low tides occur?

Max: 6, 18

Min: 12

High Tide: 6:00am, 6:00pm Low Tide: 12:00 noon

$x\text{-min}: 0$

$x\text{-max}: 24$

Scale: 1

$y\text{-min}: 0$

$y\text{-max}: 65$

Scale: 5

b) At what time(s) is the water depth 3.5 feet?

$3.5 = 35 - 28 \cos \frac{\pi}{6} t$

$1.125 = \cos \frac{\pi}{6} t$

undefined

10. The number of hours of daylight in Boston during one year is given by $y = 3\sin\left[\frac{2\pi}{365}(x - 79)\right] + 12$ where x is the number of days after January 1.

a) Within a year, when does Boston have 10.5 hours of daylight? Give your answer in days after January 1 and round to the nearest day.

$$10.5 = 3\sin\left[\frac{2\pi}{365}(x - 79)\right] + 12$$

$$-0.5 = \sin\left[\frac{2\pi}{365}(x - 79)\right]$$

S/A $\frac{\pi}{6}$
 $\frac{2\pi}{6} = \frac{2\pi}{365}(x - 79)$
 $\frac{11\pi}{6} = \frac{2\pi}{365}(x - 79)$

$x_{\min} = 0$ $y_{\min} = -2$
 $x_{\max} = 365$ $y_{\max} = 12$

$$\frac{7\pi}{6} = \frac{2\pi}{365}(x - 79)$$

$$-\frac{\pi}{6} = \frac{2\pi}{365}(x - 79)$$

$$\frac{2555}{12} = x - 79$$

$$-\frac{365}{12} = x - 79$$

$$291.917 = x$$

$$48.583 = x$$

$x = 48.58$ days after Jan 1 $x = 291.9$ days after Jan 1

b) Within a year, when does Boston have 13.5 hours of daylight? Give your answer in days after January 1 and round to the nearest day.

$$13.5 = 3\sin\left[\frac{2\pi}{365}(x - 79)\right] + 12$$

$$\frac{1}{2} = \sin\left[\frac{2\pi}{365}(x - 79)\right]$$

$\frac{\pi}{6} = \frac{2\pi}{365}(x - 79)$ $\frac{5\pi}{6} = \frac{2\pi}{365}(x - 79)$
 $\frac{365}{12} = x - 79$ $\frac{1825}{12} = x - 79$

$x = 109.417$ days after Jan 1st $x = 231.083$ days after Jan 1st

11. A ball on a spring is pulled 4 inches below its rest position and then released. After t seconds, the ball's distance, d , in inches from its resting position is given by $d = -4\cos\left(\frac{\pi}{3}t\right)$

a) Find all values of t for which the ball is 2 inches above its rest position.

$$2 = -4\cos\left(\frac{\pi}{3}t\right)$$

$$-\frac{1}{2} = \cos\left(\frac{\pi}{3}t\right)$$

S/A $\frac{2\pi}{3}$
 $\frac{4\pi}{3}$

$\frac{2\pi}{3} = \frac{\pi}{3}t$ $\frac{4\pi}{3} = \frac{\pi}{3}t$
 $2 = t$ $4 = t$

2 seconds $\pm 2\pi n$ 4 seconds $\pm 2\pi n$

b) Find all values of t for which the ball is 2 inches below its rest position.

$$-2 = -4\cos\left(\frac{\pi}{3}t\right)$$

$$\frac{1}{2} = \cos\left(\frac{\pi}{3}t\right)$$

S/A $\frac{\pi}{3}$
 $\frac{5\pi}{3}$

$\frac{\pi}{3} = \frac{\pi}{3}t$ $\frac{5\pi}{3} = \frac{\pi}{3}t$
 $1 = t$ $5 = t$

1 second $\pm 2\pi n$ 5 seconds $\pm 2\pi n$

