

Name: _____

Date: _____ Block: _____

Class Examples: Trigonometric Identities

Hr. Adv. Algebra II with Trig

Simplify the following trigonometric expressions.

1. $\sec x - \tan x \sin x$

2. $\frac{\sec^2 x}{\sec^2 x - 1}$

3. $\frac{\sec x \sin x}{\tan x + \cot x}$

4. $\csc^2 x \tan^2 x - 1$

Verify the following trigonometric expressions.

5. $\frac{1 + \cos x}{\sin x} = \csc x + \cot x$

6. $\frac{\sec x}{\cos x} - \frac{\tan x}{\cot x} = 1$

7. $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

8. $\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

9. $\frac{\tan x - 1}{\tan x + 1} = \frac{1 - \cot x}{1 + \cot x}$

10. $\frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2\sin^2 x$

11. $\frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$

12. $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$

13. $\frac{\tan x}{\sec x} + \frac{\cot x}{\csc x} = \sin x + \cos x$

14. $\frac{\sin x + \tan x}{1 + \sec x} = \sin x$

15. $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$

16. $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1 \\ 1 + \tan^2 x &= \sec^2 x \\ 1 + \cot^2 x &= \csc^2 x\end{aligned}$$

Class Examples: Trigonometric Identities (Extra Practice)

1. $\sec x - \tan x \sin x$

$$\frac{1}{\cos x} - \frac{\sin x \cdot \sin x}{\cos x}$$

$$\frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}$$

$$\frac{1 - \sin^2 x}{\cos x} \stackrel{(I)}{=} \frac{\cos^2 x}{\cos x} = \boxed{\cos x}$$

2. $\frac{\sec^2 x}{\sec^2 x - 1} = \frac{\sec^2 x}{(1 + \tan^2 x) - 1} = \frac{\sec^2 x}{\tan^2 x} = \frac{\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}} = \frac{1}{\sin^2 x} = \boxed{\csc^2 x}$

3. $\frac{\sec x \sin x}{\tan x + \cot x} = \frac{\frac{1}{\cos x} \cdot \frac{\sin x}{1}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \frac{\frac{\sin x}{\cos x} \cdot \cos x \sin x}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$

$$= \frac{\sin^2 x}{\sin^2 x + \cos^2 x} \stackrel{(I)}{=} \frac{\sin^2 x}{1} = \boxed{\sin^2 x}$$

4. $\csc^2 x \tan^2 x - 1 \rightarrow \frac{1}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} - 1 \rightarrow \frac{1}{\cos^2 x} - 1 = \sec^2 x - 1$

$$\downarrow (I)$$

$$\boxed{\tan^2 x}$$

5. $\frac{1 + \cos x}{\sin x} = \csc x + \cot x$

$$\downarrow$$

$$\frac{1}{\sin x} + \frac{\cos x}{\sin x}$$

$$\csc x + \cot x \quad \checkmark$$

$$6. \frac{\sec x}{\cos x} - \frac{\tan x}{\cot x} = 1$$

$$\frac{\frac{1}{\cos x}}{\cos x} - \frac{\frac{\sin x}{\cos x}}{\frac{1}{\sin x}} = \frac{1}{\cos^2 x} - \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$= \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \sin^2 x}{\cos^2 x} \stackrel{(I)}{=} \frac{\cos^2 x}{\cos^2 x} = 1$$

$$7. \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$$

$$(I) \downarrow$$

$$(1 - \sin^2 x) - \sin^2 x$$

$$1 - 2\sin^2 x \quad \checkmark$$

$$8. \tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$$

$$(I) \downarrow$$

$$(\sec^2 x - 1) \sin^2 x$$

$$(\sec^2 x - 1) \sin^2 x \rightarrow \sec^2 x \sin^2 x - \sin^2 x \rightarrow \frac{1}{\cos^2 x} \cdot \frac{\sin^2 x}{1} - \sin^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} - \sin^2 x$$

$$\tan^2 x - \sin^2 x \quad \checkmark$$

$$9. \frac{\tan x - 1}{\tan x + 1} = \frac{1 - \cot x}{1 + \cot x}$$

$$\downarrow$$

$$\frac{\frac{\sin x}{\cos x} - \frac{\cos x}{\cos x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}} = \frac{\frac{\sin x - \cos x}{\cos x}}{\frac{\sin x + \cos x}{\cos x}} = \frac{\sin x - \cos x}{\sin x + \cos x} \cdot \frac{\frac{1}{\sin x}}{\frac{1}{\sin x}}$$

$$= \frac{\frac{\sin x}{\sin x} - \frac{\cos x}{\sin x}}{\frac{\sin x}{\sin x} + \frac{\cos x}{\sin x}} = \frac{1 - \cot x}{1 + \cot x} \quad \checkmark$$

$$10. \frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2\sin^2 x$$

$$\begin{aligned} \text{I } \left(\frac{1 - \tan^2 x}{\sec^2 x} \right) &\rightarrow \frac{1}{\sec^2 x} - \frac{\tan^2 x}{\sec^2 x} \rightarrow \frac{1}{\cos^2 x} - \frac{\frac{\sin^2 x}{\cancel{\cos^2 x}}}{\frac{1}{\cancel{\cos^2 x}}} \\ &\qquad \qquad \qquad \downarrow \\ &\qquad \qquad \qquad \cos^2 x - \sin^2 x \\ &\qquad \qquad \qquad \downarrow \\ &\qquad \qquad \qquad (1 - \sin^2 x) - \sin^2 x \\ &\qquad \qquad \qquad 1 - 2\sin^2 x \checkmark \end{aligned}$$

$$11. \frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$$

$$\begin{aligned} &\downarrow \\ &\frac{\cos x + 1}{\sin x (\sin^2 x)} \rightarrow \frac{\cos x + 1}{\sin x (1 - \cos^2 x)} \rightarrow \frac{\cancel{\cos x + 1}}{\sin x (1 - \cos x) \cancel{(1 + \cos x)}} = \frac{1}{\sin x (1 - \cos x)} \\ &\qquad \qquad \qquad \swarrow \qquad \qquad \searrow \\ &\qquad \qquad \qquad \text{factor (diff. of squares)} \\ &\frac{1}{\sin x} \cdot \frac{1}{1 - \cos x} \rightarrow \csc x \cdot \frac{1}{1 - \cos x} = \frac{\csc x}{1 - \cos x} \checkmark \end{aligned}$$

$$12. \csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$$

↑
diff of squares

$$\begin{aligned} &(\csc^2 x - \cot^2 x)(\csc^2 x + \cot^2 x) \\ \text{I } \downarrow & \\ &((1 + \cot^2 x) - \cot^2 x)(\csc^2 x + \cot^2 x) \\ &(1)(\csc^2 x + \cot^2 x) \\ &\csc^2 x + \cot^2 x \checkmark \end{aligned}$$

$$13. \frac{\tan x}{\sec x} + \frac{\cot x}{\csc x} = \sin x + \cos x$$

$$\frac{\sin x}{\cancel{\cos x}} + \frac{\cos x}{\cancel{\sin x}}$$

$$\frac{1}{\cos x} + \frac{1}{\sin x}$$

$$\frac{\sin x}{1} + \frac{\cos x}{1}$$

$$\sin x + \cos x \checkmark$$

$$14. \frac{\sin x + \tan x}{1 + \sec x} = \sin x$$

$$\frac{\sin x + \sin x}{1 + \frac{1}{\cos x}}$$

$$\frac{\sin x + \sin x}{1 + \frac{1}{\cos x}}$$

$$\cdot \frac{\cos x}{\cos x}$$

$$\rightarrow \frac{\sin x \cos x + \sin x}{\cos x + 1}$$

$$\rightarrow \frac{\sin x (\cos x + 1)}{\cos x + 1}$$

$$= \sin x \checkmark$$

$$15. (\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$$

$$\sin^2 x + 2\sin x \cos x + \cos^2 x + \sin^2 x - 2\sin x \cos x + \cos^2 x$$

$$\underbrace{\sin^2 x + \cos^2 x}_{(I)} + \underbrace{\sin^2 x + \cos^2 x}_{(I)}$$

$$1 + 1$$

$$2 \checkmark$$

$$16. (\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$$

$$\sin x \tan x + \sin x \cot x + \cos x \tan x + \cos x \cot x$$

$$\frac{\sin x \cdot \sin x}{\cos x} + \frac{\sin x \cdot \cos x}{\sin x} + \frac{\cos x \cdot \sin x}{\cos x} + \frac{\cos x \cdot \cos x}{\sin x}$$

$$\frac{\sin^2 x}{\cos x} + \cos x + \sin x + \frac{\cos^2 x}{\sin x}$$

$$\frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x} \rightarrow \frac{\sin^2 x + \cos^2 x}{\cos x} + \frac{\sin^2 x + \cos^2 x}{\sin x}$$

$$\frac{1}{\cos x} + \frac{1}{\sin x} = \sec x + \csc x \checkmark$$