

Homework: Trigonometric Identities (1)

1) $\sin x + \cos x \cot x$

$$\sin x + \cos x \frac{\cos x}{\sin x}$$

$$\sin x + \frac{\cos^2 x}{\sin x}$$

$$\frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x}$$

$$\boxed{\frac{1}{\sin x}}$$

2) $(\sec x - \tan x)(\csc x + 1)$

$$\sec x \csc x + \sec x - \tan x \csc x - \tan x$$

$$\frac{1}{\cos x} \cdot \frac{1}{\sin x} + \frac{1}{\cos x} - \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} - \frac{\sin x}{\cos x}$$

$$\frac{1}{\cos x} \cdot \frac{1}{\sin x} + \frac{1}{\cos x} - \frac{1}{\cos x} - \frac{\sin x}{\cos x}$$

$$\frac{1}{\cos x} \left(\frac{1}{\sin x} + 1 - 1 - \sin x \right)$$

$$\frac{1}{\cos x} \left(\frac{1}{\sin x} - \sin x \right)$$

$$\frac{1}{\cos x} \left(\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} \right)$$

$$\frac{1}{\cos x} \left(\frac{1 - \sin^2 x}{\sin x} \right)$$

$$\frac{1}{\cos x} \cdot \frac{\cos^2 x}{\sin x} = \frac{\cos x}{\sin x} = \boxed{\cot x}$$

$$\begin{aligned}
 3) \quad \frac{1 + \sec x}{\sin x + \tan x} &= \frac{1 + \frac{1}{\cos x}}{\sin x + \frac{\sin x}{\cos x}} = \frac{\frac{\cos x}{\cos x} + \frac{1}{\cos x}}{\frac{\sin x \cos x}{\cos x} + \frac{\sin x}{\cos x}} = \frac{\cos x + 1}{\cos x} \\
 &= \frac{\cos x + 1}{\sin x \cos x + \sin x} = \frac{\cos x + 1}{\sin x (\cos x + 1)} = \frac{1}{\sin x}
 \end{aligned}$$

$$\begin{aligned}
 4) \quad \frac{1}{\tan x + \cot x} &= \frac{1}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \frac{1}{\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}} = \frac{\cos x \sin x}{\sin^2 x + \cos^2 x} \\
 &= \boxed{\cos x \sin x}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad \csc x - \sin x &= \cot x \cos x \\
 \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} &= \frac{1 - \sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x} = \frac{\cos x \cos x}{\sin x} = \cot x \cos x \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 6) \quad \frac{\sec x + \csc x}{\tan x + \cot x} &= \frac{\sin x + \cos x}{\sin x \cos x} \\
 \frac{\frac{1}{\cos x} + \frac{1}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} &= \frac{\frac{\sin x + \cos x}{\cos x \sin x}}{\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}} = \frac{\sin x + \cos x}{\sin^2 x + \cos^2 x} = \frac{\sin x + \cos x}{\sin x \cos x} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 7) \quad \frac{\tan^2 x - \sin^2 x}{\cos^2 x} &= \frac{\tan^2 x \sin^2 x}{\cos^2 x} \\
 \frac{\frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x \cos^2 x}{\cos^2 x}}{\cos^2 x} &= \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x} = \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x} \\
 &= \frac{\sin^2 x (\sin^2 x)}{\cos^2 x} = \frac{\sin^2 x \cdot \sin^2 x}{\cos^2 x} = \tan^2 x \sin^2 x \quad \checkmark
 \end{aligned}$$

$$8) \frac{\cot x - \tan x}{\sin x + \cos x} = \csc x - \sec x$$

$$\frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}{\sin x + \cos x} = \frac{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}}{\sin^2 x \cos x + \sin x \cos^2 x} = \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos x + \sin x \cos^2 x}$$

$$\frac{(\cos x + \sin x)(\cos x - \sin x)}{\sin x \cos x (\sin x + \cos x)} = \frac{\cos x - \sin x}{\sin x \cos x} = \frac{\cos x}{\sin x \cos x} - \frac{\sin x}{\sin x \cos x}$$

$$= \frac{1}{\sin x} - \frac{1}{\cos x}$$

$$= \csc x - \sec x \quad \checkmark$$

$$9) \sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$$

$$\frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{(\sin^2 x - \cos^2 x)(1)}$$

$$= \sin^2 x - \cos^2 x \quad \checkmark$$

$$10) \frac{\tan^2 x}{\sec x + 1} = \frac{1 - \cos x}{\cos x}$$

$$\frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos x} + \frac{\cos x}{\cos x}} = \frac{\frac{1 - \cos^2 x}{\cos^2 x}}{\frac{\cos x + \cos^2 x}{\cos^2 x}} = \frac{1 - \cos^2 x}{\cos x + \cos^2 x} = \frac{(1 - \cos x)(1 + \cos x)}{\cos x(1 + \cos x)}$$

$$= \frac{1 - \cos x}{\cos x} \quad \checkmark$$