

Homework : Trigonometric Identities (2)

$$1) \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = \frac{\sin x}{\frac{1}{\sin x}} + \frac{\cos x}{\frac{1}{\cos x}}$$

$$\sin^2 x + \cos^2 x = \boxed{1}$$

$$2) \tan x \cos x \csc x$$

$$\frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} \cdot \frac{1}{\sin x} = \boxed{1}$$

$$3) \frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = \frac{(1 + \sin x)^2 + \cos^2 x}{\cos x (1 + \sin x)}$$

$$\frac{1 + \sin x + \sin x + \sin^2 x + \cos^2 x}{\cos x (1 + \sin x)} = \frac{1 + 2\sin x + 1}{\cos x (1 + \sin x)}$$

$$= \frac{2 + 2\sin x}{\cos x (1 + \sin x)} = \frac{2(1 + \sin x)}{\cos x (1 + \sin x)} = \boxed{\frac{2}{\cos x}}$$

$$4) \frac{\cos x}{\sec x + \tan x} = \frac{\cos x}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} = \frac{\cos x}{\frac{1 + \sin x}{\cos x}} = \frac{\cos x \cdot \cos x}{1 + \sin x}$$

$$= \frac{\cos^2 x}{1 + \sin x} = \frac{1 - \sin^2 x}{1 + \sin x} = \frac{(1 - \sin x)(1 + \sin x)}{1 + \sin x} = \boxed{1 - \sin x}$$

$$5) \frac{\cos x}{\sec x \sin x} = \csc x - \sin x$$

$$\frac{\cos x}{\frac{1}{\cos x} \cdot \sin x} = \frac{\cos x}{\frac{\sin x}{\cos x}} = \cos x \cdot \frac{\cos x}{\sin x} = \frac{\cos^2 x}{\sin x}$$

$$= \frac{1 - \sin^2 x}{\sin x} = \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} = \csc x - \sin x \quad \checkmark$$

$$6) \frac{\sin x}{\sin x + \cos x} = \frac{\tan x}{1 + \tan x}$$

$$\begin{aligned} \frac{\frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} &= \frac{\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}} = \frac{\frac{\sin x}{\cos x}}{\frac{\cos x + \sin x}{\cos x}} \\ &= \frac{\sin x}{\cos x + \sin x} \quad \checkmark \end{aligned}$$

$$7) \cos^2 x \csc x - \csc x = -\sin x$$

$$\cos^2 x \cdot \frac{1}{\sin x} - \frac{1}{\sin x}$$

$$\frac{1}{\sin x} (\cos^2 x - 1) \rightarrow \frac{-1}{\sin x} (-\cos^2 x + 1) = \frac{-1}{\sin x} (\sin^2 x)$$

$$= \frac{-\sin^2 x}{\sin x} = -\sin x \quad \checkmark$$

$$8) \frac{\cos^2 x - \tan^2 x}{\sin^2 x} = \cot^2 x - \sec^2 x$$

$$\begin{aligned} \frac{\cos^2 x}{\sin^2 x} - \frac{\tan^2 x}{\sin^2 x} &= \cot^2 x - \frac{\frac{\sin^2 x}{\cos^2 x}}{\sin^2 x} = \cot^2 x - \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x} \\ &= \cot^2 x - \frac{1}{\cos^2 x} \\ &= \cot^2 x - \sec^2 x \quad \checkmark \end{aligned}$$

$$9) \frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = 1$$

$$\frac{\cos x}{\frac{1}{\cos x}} + \frac{\sin x}{\frac{1}{\sin x}}$$

$$\cos^2 x + \sin^2 x = 1 \quad \checkmark$$

$$10) \frac{\sec x - \cos x}{\sec x} = \sin^2 x$$

$$\frac{\frac{1}{\cos x} - \cos x}{\frac{1}{\cos x}} = \frac{1 - \cos^2 x}{1} = \sin^2 x \quad \checkmark$$