Name: Answers Block: ____

Date: ______________

Homework: Angle Sum and Difference Formulas **Honors PreCalculus**

Find the exact value of each expression.

1. $\sin\frac{5\pi}{12} = \sin\left(\frac{3\pi}{12}+\frac{2\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6}+\cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

2. $\cos 165^\circ = \cos(45+120)$

$= \cos 45\cos 120 - \sin 45 \sin 120$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$

3. $\tan\frac{7\pi}{12} = \tan\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right)$

$= \tan\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$\frac{\sin\frac{\pi}{4}\cos\frac{\pi}{3}+\cos\frac{\pi}{4}\sin\frac{\pi}{3}}{\cos\frac{\pi}{4}\cos\frac{\pi}{3}-\sin\frac{\pi}{4}\sin\frac{\pi}{3}} = \frac{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}$

$= \frac{\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}}{\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}} = \frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}-\sqrt{6}}\cdot\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}+\sqrt{6}} = \frac{8+2\sqrt{12}}{2-6} = \frac{8+4\sqrt{3}}{-4}$ ($4\cdot 3$)

$= -2-\sqrt{3}$

4. $\sin\frac{17\pi}{12} = \sin\left(\frac{3\pi}{12}+\frac{14\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4}+\frac{7\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{7\pi}{6}+\cos\frac{\pi}{4}\sin\frac{7\pi}{6}$

$\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right)$

$\frac{-\sqrt{6}-\sqrt{2}}{4}$

5. $\sec\left(-\frac{\pi}{12}\right)$

$\cos\left(\frac{3\pi}{12}-\frac{4\pi}{12}\right) = \cos\left(\frac{\pi}{4}-\frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3}+\sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}+\sqrt{6}}{4}$

$\sec\left(-\frac{\pi}{12}\right) = \frac{4}{\sqrt{2}+\sqrt{6}}\cdot\frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}}$

$= \frac{4\sqrt{2}-4\sqrt{6}}{2-\sqrt{12}+\sqrt{12}-6} = \frac{4\sqrt{2}-4\sqrt{6}}{-4} = \frac{\sqrt{2}-\sqrt{6}}{-1} = -(\sqrt{2}-\sqrt{6})$ or $-\sqrt{2}+\sqrt{6}$

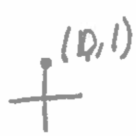
6. $\cos 70^\circ\cos 20^\circ - \sin 70^\circ \sin 20^\circ$

(cosine formula)

$= \cos(70+20)$

$= \cos(90)$

$= 0$



$(0,1)$

Verify the following trigonometric identities.

7. $\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$

$\sin\frac{\pi}{2}\cos\theta + \cos\frac{\pi}{2}\sin\theta$

$(1)\cos\theta + (0)\sin\theta$

$\cos\theta$ ✓

8. $\frac{\sin(\alpha+\beta)}{\sin\alpha\cos\beta} = 1 + \cot\alpha\tan\beta$

$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta}$

$= \frac{\sin\alpha\cos\beta}{\sin\alpha\cos\beta} + \frac{\cos\alpha\sin\beta}{\sin\alpha\cos\beta}$

$= 1 + \frac{\cos\alpha}{\sin\alpha}\cdot\frac{\sin\beta}{\cos\beta}$

$= 1 + \cot\alpha\tan\beta$ ✓

9. $\sec(\alpha+\beta) = \frac{\csc\alpha\csc\beta}{\cot\alpha\cot\beta - 1}$

$\frac{1}{\cos(\alpha+\beta)} = \frac{1}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$

$\frac{\frac{1}{\sin\alpha}\cdot\frac{1}{\sin\beta}}{\frac{\cos\alpha}{\sin\alpha}\cdot\frac{\cos\beta}{\sin\beta} - 1}$

$\frac{\frac{1}{\sin\alpha\sin\beta}}{\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} + \frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}} = \frac{\frac{1}{\sin\alpha\sin\beta}}{\frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\sin\beta}}$

$= \frac{1}{\sin\alpha\sin\beta}\cdot\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta} = \frac{1}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$

$= \frac{1}{\cos(\alpha+\beta)} = \sec(\alpha+\beta)$ ✓

10. $\frac{\cot x\sec x}{\csc x} = 1$

$\frac{\frac{\cos x}{\sin x}\cdot\frac{1}{\cos x}}{\frac{1}{\sin x}} = \frac{\frac{1}{\sin x}}{\frac{1}{\sin x}} = 1$ ✓

11. $\frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$

$\frac{\sin\alpha}{\sin\alpha}\cdot\frac{1-\cos\alpha}{\sin\alpha}$

$\frac{\sin\alpha(1-\cos\alpha)}{\sin^2\alpha} \rightarrow \frac{\sin\alpha(1-\cos\alpha)}{1-\cos^2\alpha}$

$\frac{\sin\alpha(1-\cancel{\cos\alpha})}{(1-\cancel{\cos\alpha})(1+\cos\alpha)} = \frac{\sin\alpha}{1+\cos\alpha}$ ✓

12. $\ln e^{\tan^2 x - \sec^2 x} = -1$

logarithm exponent rule (exponent comes to the front... like $\log x^2 = 2\log x$)

$(\tan^2 x - \sec^2 x)\ln e \longrightarrow \ln e = x$, $e^x = e$, $x = 1$

$[\tan^2 x - (1+\tan^2 x)][1]$

$\cancel{\tan^2 x} - 1 - \cancel{\tan^2 x}$

-1 ✓