Homework: Composition of Functions

Textbook Page 256-258 Problems 8(a, c, e), 10(a, d), 17(a, d), 38, 40, 44 d

8a) $f(g(1))$ $g(1)=0$
$f(0)=\boxed{5}$

c) $g(f(2))$ $f(2)=1$
$g(1)=\boxed{0}$

e) $g(g(1))$ $g(1)=0$
$g(0)=\boxed{1}$

10a) $g(f(1))$ $f(1)=-1$
$g(-1)=\boxed{3}$

d) $f(g(2))$ $g(2)=2$
$f(2)=\boxed{-2}$

17a) $f(g(4))$ where $f(x)=|x|$ and $g(x)=\frac{1}{x^2+1}$
$g(4)=\frac{1}{17}$
$f\left(\frac{1}{17}\right)=\left|\frac{1}{17}\right|=\boxed{\frac{1}{17}}$

d) $g(g(0))$
$g(0)=\frac{1}{0^2+1}=\frac{1}{1}=1$
$g(1)=\frac{1}{1^2+1}=\boxed{\frac{1}{2}}$

38) $f(x)=\frac{x}{x+3}$ $g(x)=\frac{2}{x}$

a) $f(g(x))=\frac{\frac{2}{x}}{\frac{2}{x}+3}\cdot\frac{x}{x}=\boxed{\frac{2}{2+3x}}$

D: $x\neq 0, -\frac{2}{3}$
$(-\infty,-\frac{2}{3})\cup(-\frac{2}{3},0)\cup(0,\infty)$

b) $g(f(x))=\frac{2}{\frac{x}{x+3}}=2\cdot\frac{x+3}{x}=\boxed{\frac{2(x+3)}{x}}$

D: $x\neq 0, x\neq -3$
$(-\infty,-3)\cup(-3,0)\cup(0,\infty)$

c) $f(f(x))=\frac{\frac{x}{x+3}}{\frac{x}{x+3}+3}\cdot\frac{x+3}{x+3}=\frac{x}{x+3(x+3)}$

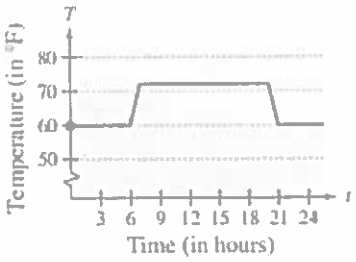
$=\boxed{\frac{x}{4x+9}}$

D: $x\neq -3, -\frac{9}{4}$
$(-\infty,-3)\cup(-3,-\frac{9}{4})\cup(-\frac{9}{4},\infty)$

d) $g(g(x))=\frac{2}{\frac{2}{x}}=2\cdot\frac{x}{2}=\frac{2x}{2}=\boxed{x}$

D: $x\neq 0$
D: $(-\infty,0)\cup(0,\infty)$

1. An electronically controlled thermostat in a home is programmed to lower the temperature automatically during the night. The temperature in the house T (in degrees Fahrenheit) is given in terms of t, the time in hours on a 24-hour clock as shown in the diagram.



a) Explain why T is a function of t.

Temperature is dependent on time

b) Approximate $T(4)$ and $T(15)$.

$T(4) = 60$

$T(15) = 72$

c) The thermostat is reprogrammed to produce a temperature H for which $H(t) = T(t-1)$. How does this change the temperature?

The entire graph is shifted to the right 1 unit, so all temperatures will occur 1 hour later.

d) The thermostat is reprogrammed to produce a temperature H for which $H(t) = T(t) - 1$. How does this change the temperature?

The entire graph is shifted down 1 unit, so all temperatures will lower 1°F

e) Write a piece-wise defined function that represents the graph.

$$H(t) = \begin{cases} 60, & 0 \le t \le 6,\ 21 < t \le 24 \\ 12t - 12, & 6 < t \le 7 \\ 72, & 7 < t \le 20 \\ -12t + 312, & 20 < t \le 21 \end{cases}$$

$(6,60)$ $(7,12)$ $m = 12$

$T - 60 = 12(t-6)$

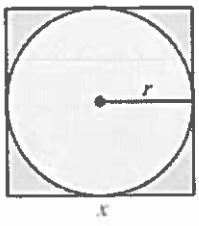
$T = 12t - 72 + 60$ $T = 12t - 12$

$(20,72)$ $(21,60)$ $m = -12$

$T - 60 = -12(t-21)$

$T = -12t + 252 + 60$ $T = -12t + 312$

2. A square concrete foundation is prepared as a base for a cylindrical tank. See the diagram below.



a) Write the radius r of the tank as a function of the length x of the sides of the square.

$r(x) = \frac{1}{2}x$

b) Write the area A of the circular base of the tank as a function of the radius r.

$A(r) = \pi r^2$

c) Find and interpret $(A \circ r)(x)$.

$A(r(x)) = \pi\left(\frac{1}{2}x\right)^2 = \frac{1}{4}\pi x^2$

Area when radius is $\frac{1}{2}x$

3. You are a sales representative for a clothing manufacturer. You are paid an annual salary, plus a bonus of 3% of your sales over \$500,000. Consider the two functions given by $f(x) = x - 500{,}00$ and $g(x) = 0.03x$. If x is greater than \$500,000, which of the following represents your bonus? $f(g(x))$ or $g(f(x))$ Explain your reasoning.

$g(f(x))$ because you find $f(x)$ first which represents amount over 500,000 and then multiply by 3%

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40) $f(x)=\sqrt{x-2}$ $g(x)=1-2x$

a) $f(g(x)) = \sqrt{(1-2x)-2}$

$= \boxed{\sqrt{-2x-1}}$

D: $-2x-1 \geq 0$

$-2x \geq 1$

$x \leq -\frac{1}{2}$

$\boxed{(-\infty, -\frac{1}{2}]}$

b) $g(f(x)) = 1-2\sqrt{x-2}$

D: $x-2 \geq 0$

$x \geq 2$

$\boxed{[2, \infty)}$

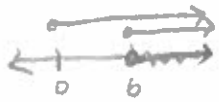
c) $f(f(x)) = \boxed{\sqrt{\sqrt{x-2}-2}}$

D: $x-2 \geq 0$ $\sqrt{x-2}-2 \geq 0$

$x \geq 0$ and $\sqrt{x-2} \geq 2$

$x-2 \geq 4$

$x \geq 6$



$\boxed{D: [6, \infty)}$

d) $g(g(x)) = 1-2(1-2x)$

$1-2+4x$

$-1+4x$

D: ARN

$(-\infty, \infty)$

44d) $f(x) = \frac{2x-1}{x-2}$, $g(x) = \frac{x+4}{2x-5}$

$g(g(x)) = \frac{\frac{x+4}{2x-5}+4}{2\left(\frac{x+4}{2x-5}\right)-5}$

$\frac{2x-5}{2x-5} \cdot$

$= \frac{x+4+4(2x-5)}{2(x+4)-5(2x-5)}$

$= \frac{x+4+8x-20}{2x+8-10x+25}$

$= \frac{9x-16}{-8x+33}$

$= \boxed{\frac{9x-16}{-(8x-33)}}$

D: $x \neq \frac{5}{2}, \frac{33}{8}$

$\boxed{(-\infty, \frac{5}{2}) \cup (\frac{5}{2}, \frac{33}{8}) \cup (\frac{33}{8}, \infty)}$