Hrs Precalculus

Quarter 1 Exam Studyguide

(1) a) $\dfrac{(2x^4y^{-\frac{4}{5}})^3(8y^2)^{\frac{2}{3}}}{(8x^{12}y^{-\frac{12}{5}})(4y^{\frac{4}{3}})}$

$32x^{12}y^{-\frac{16}{15}}$

$\boxed{\dfrac{32x^{12}}{y^{\frac{16}{15}}}}$

b) $\left(\dfrac{a^4b^{\frac{2}{5}}}{c^{-2}d}\right)^2\left(\dfrac{a^{\frac{3}{2}}d^4}{bc^{-\frac{1}{2}}}\right)^{-1}$

$\left(\dfrac{a^8b^{\frac{4}{5}}}{c^{-4}d^2}\right)\left(\dfrac{bc^{-\frac{1}{2}}}{a^{\frac{3}{2}}d^4}\right)$

$\dfrac{a^8b^{\frac{9}{5}}c^{-\frac{1}{2}}}{a^{\frac{3}{2}}c^{-4}d^6}$ $\boxed{\dfrac{a^{\frac{13}{2}}b^{\frac{9}{5}}c^{\frac{7}{2}}}{d^6}}$

c) $(3x+2y^2)(3x-2y^2)$

$\boxed{9x^2-4y^4}$

d) $\dfrac{x^{\frac{1}{3}}y^{\frac{2}{3}}}{(xy)^{-\frac{2}{3}}} = x^{\frac{1}{3}}y^{\frac{2}{3}}x^{\frac{2}{3}}y^{\frac{2}{3}} = \boxed{xy^{\frac{4}{3}}}$

(2) a) $(z^2+1)^2-7(z^2+1)+10 \quad a=(z^2+1)$

$a^2-7a+10$

$(a-5)(a-2)$

$[(z^2+1)-5][(z^2+1)-2]$

$[z^2-4][z^2-1]$

$\boxed{(z+2)(z-2)(z+1)(z-1)}$

b) $12x^2(4x-1)^3(2x-1)^2+4x(4x-1)^4(2x-1)$

$4x(4x-1)^3(2x-1)[3x(2x-1)+(4x-1)]$

$4x(4x-1)^3(2x-1)[6x^2-3x+4x-1]$

$4x(4x-1)^3(2x-1)(6x^2+x-1)$

$\boxed{4x(4x-1)^3(2x-1)(2x+1)(3x-1)}$

$\frac{3}{3}\ x^{-2} = -6$

$\frac{3}{3} + -2 = 1$

$\frac{6}{3} = \frac{2}{1} \quad \frac{6}{-2} = \frac{3}{-1}$

② c) $4x^4 - 13x^2 + 9$

$-9 \times -4 = 36$

$-9 + -4 = -13$

$\frac{4}{-9} \quad \frac{4}{-4} = \frac{1}{-1}$

$(4x^2 - 9)(x^2 - 1)$

$\boxed{(2x+3)(2x-3)(x+1)(x-1)}$

d) $8x^5 - 32x^3 + 27x^2 - 108$

$8x^3(x^2 - 4) + 27(x^2 - 4)$

$(8x^3 + 27)(x^2 - 4)$

$\boxed{(2x+3)(4x^2 - 6x + 9)(x+2)(x-2)}$

e) $(x^2+4)^2 - 13(x^2+4) + 40 \qquad a = (x^2+4)$

$a^2 - 13a + 40$

$(a-8)(a-5)$

$[(x^2+4) - 8][(x^2+4) - 5]$

$(x^2 - 4)(x^2 - 1)$

$\boxed{(x+2)(x-2)(x+1)(x-1)}$

f) $8x^3 - 27$

$\boxed{(2x-3)(4x^2 + 6x + 9)}$

g) $9x^2 - 4y^2$

$\boxed{(3x - 2y)(3x + 2y)}$

h) $5x^6(x+3)^4 - 15x^7(x+3)^3$

$5x^6(x+3)^3[(x+3) - 3x]$

$5x^6(x+3)^3(-2x+3)$

$\boxed{-5x^6(x+3)^3(2x-3)}$

③ $3\sqrt{4x-1}+5=6x-4$

$3\sqrt{4x-1}=6x-9$

$(\sqrt{4x-1})^2=(2x-3)^2$

$4x-1=4x^2-12x+9$

$0=4x^2-16x+10$

$x=\frac{16\pm\sqrt{(-16)^2-4(4)(10)}}{2(4)}$

$x=\frac{16\pm\sqrt{96}}{8}$

$x=\frac{16\pm4\sqrt{6}}{8}$

$\boxed{x=\frac{4\pm\sqrt{6}}{2}}$

④ $2x+6y=17$

$6y=-2x+17$

$y=-\frac{1}{3}x+\frac{17}{6}$

$m=3\ (-2,3)$

$y-3=3(x+2)$

$\boxed{y=3x+9}$

⑤ $3x+11y+14=0$

$11y=-3x-14$

$y=-\frac{3}{11}x-\frac{14}{11}$

$m=-\frac{3}{11}\ (-\frac{1}{2},6)$

$y-6=-\frac{3}{11}(x+\frac{1}{2})$

$\boxed{y=-\frac{3}{11}x+\frac{129}{22}}$

⑥ $(-2,3)\ (7,-1)$

$m=\frac{4}{-9}$

$\perp m=\frac{9}{4}\ (-5,7)$

$y-7=\frac{9}{4}(x+5)$

$\boxed{y=\frac{9}{4}x+\frac{73}{4}}$

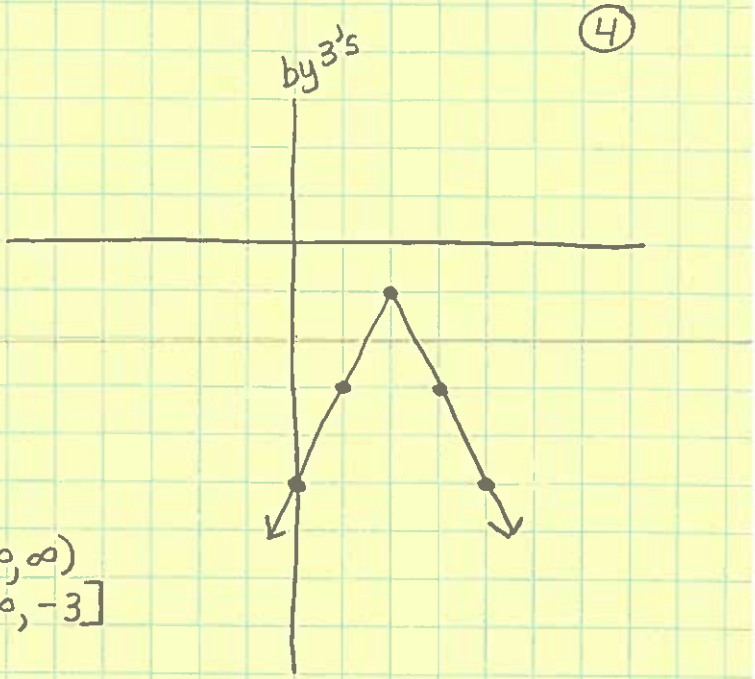
⑦ See graph paper

(7) a) $2-y=6|x-2|+5$

$-y=6|x-2|+3$

$y=-6|x-2|-3$

$y=\|x\|$	$x+2, -6y-3$
$(-2,2)$	$(0,-15)$
$(-1,1)$	$(1,-9)$
$(0,0)$	$(2,-3)$
$(1,1)$	$(3,-9)$
$(2,2)$	$(4,-15$



D: $(-\infty,\infty)$
R: $(-\infty,-3]$

Description: Reflect over x-axis, Vertical Stretch by 6
Right 2 down 3

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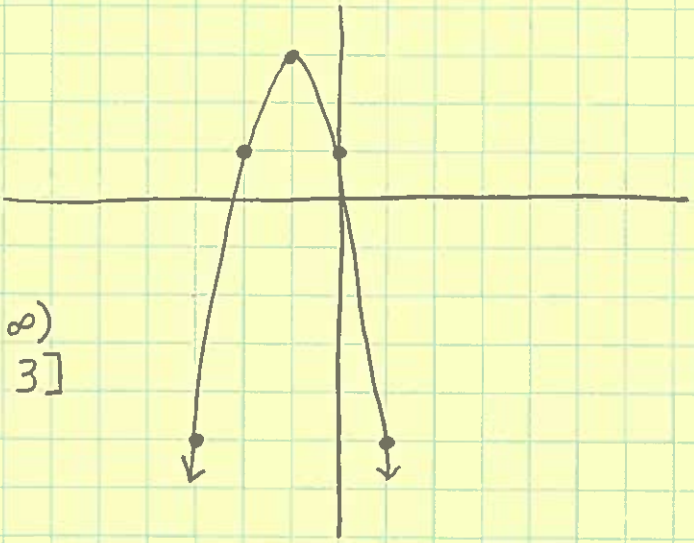
b) $y-3=-2(x+1)^2$

$y=-2(x+1)^2+3$

$y=x^2$	$x-1, -2y+3$
$(-2,4)$	$(-3,-5)$
$(-1,1)$	$(-2,1)$
$(0,0)$	$(-1,3)$
$(1,1)$	$(0,1)$
$(2,4)$	$(1,-5)$



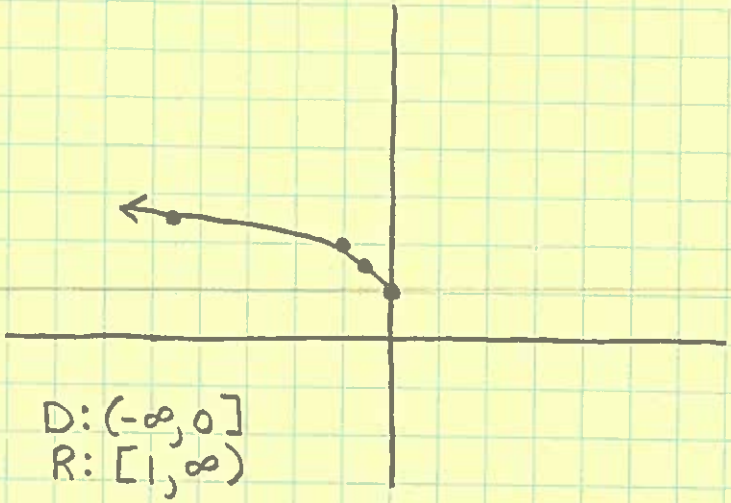
D: $(-\infty,\infty)$
R: $(-\infty,3]$

Description: Reflect over x-axis, Vertical stretch by 2
Left 1, up 3

c) $y+4=\frac{1}{2}\sqrt{-2x}+5$

$y=\frac{1}{2}\sqrt{-2x}+1$

$y=\sqrt{x}$	$-\frac{1}{2}x,\ \frac{1}{2}y+1$
(0, 0)	(0, 1)
(1, 1)	(-½, 1.5)
(4, 2)	(-2, 2)
(9, 3)	(-4.5, 2.5)



D: $(-\infty, 0]$
R: $[1, \infty)$

Description: Reflect over y-axis, Horizontal Compress by $\frac{1}{2}$
Vertical Compress by $\frac{1}{2}$, up 1

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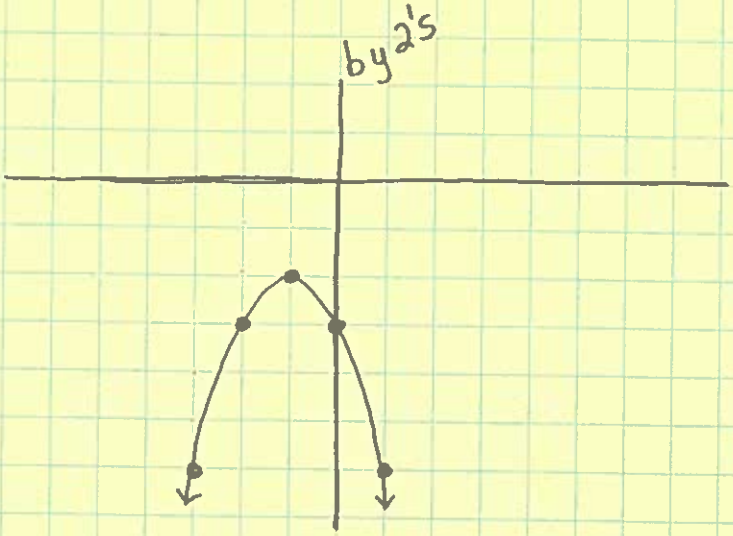
d) $f(x)=-2(x+1)^2-4$

$f(x)=x^2$	$x-1,\ -2y-4$
(-2, 4)	(-3, -12)
(-1, 1)	(-2, -6)
(0, 0)	(-1, -4)
(1, 1)	(0, -6)
(2, 4)	(1, -12)



Description: Reflect over x-axis, Vertical Stretch by 2,
left 1, down 4

(8) a) $y = 7x^5 + x$

X-axis $-y = 7x^5 + x$

$y = -7x^5 - x$ No

y-axis $y = 7(-x)^5 + (-x)$

$y = -7x^5 - x$ No

Origin $-y = 7(-x)^5 + (-x)$

$-y = -7x^5 - x$

$y = 7x^5 + x$ yes

b) $y^4 - 8x^3 = 9x^2 + 7|y| + 3$

X-axis $(-y)^4 - 8x^3 = 9x^2 + 7|-y| + 3$

$y^4 - 8x^3 = 9x^2 + 7|y| + 3$ yes

y-axis $y^4 - 8(-x)^3 = 9(-x)^2 + 7|y| + 3$

$y^4 + 8x^3 = 9x^2 + 7|y| + 3$ No

Origin $(-y)^4 - 8(-x)^3 = 9(-x)^2 + 7|-y| + 3$

$y^4 + 8x^3 = 9x^2 + 7|y| + 3$ No

(9) a) $y = 7x^4 - 9x^2 + 1$ Even

b) $f(x) = 6x^3 - 9x + 5$ Neither

(10) a) $f(x) = \dfrac{\sqrt{x^2-5x-14}}{\sqrt{(x-7)(x+2)}}$

$(x-7)(x+2) \geq 0$

D: $(-\infty, -2] \cup [7, \infty)$

$\{x \mid x \leq -2 \text{ or } x \geq 7\}$

b) $f(x) = \dfrac{2}{x+4}$ $\quad x+4 \neq 0$, $x \neq -4$

$\{x \mid x \neq -4\}$

$(-\infty, -4) \cup (-4, \infty)$

c) $f(x) = \sqrt{5x-1}$ $\quad 5x-1 \geq 0$, $x \geq \frac{1}{5}$

$\{x \mid x \geq \frac{1}{5}\}$

$[\frac{1}{5}, \infty)$

d) $f(x) = \dfrac{x^2+9x}{x^2-3x-40} = \dfrac{x^2+9x}{(x-8)(x+5)}$

$x \neq 8, -5$

$\{x \mid x \neq -5, 8\}$

$(-\infty, -5) \cup (-5, 8) \cup (8, \infty)$

e) $y = \dfrac{3x^2-13x+4}{(x+4)(x-4)}$ $\quad x \neq \pm 4$

$\{x \mid x \neq \pm 4\}$

$(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

(11) $y = \sqrt{25-x^2}$ Semicircle with $r = 5$

yes, function

Not 1-1

D: $[-5, 5]$

R: $[0, 5]$

(12) $y = 3x^3 - 4x^2 + 7x - 8$

yes, function
Not 1-1
D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

xint

$0 = 3x^3 - 4x^2 + 7x - 8$

$x = 1.217_{m3}$ Calculator

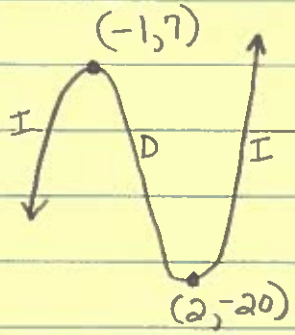
yint

$y = 3(0)^3 - 4(0)^2 + 7(0) - 8$

$y = -8$

(13) See Graph Paper

(14) $f(x) = 2x^3 - 3x^2 - 12x$



use calculator to find max and min points

Increasing
$(-\infty, -1) \cup (2, \infty)$

Decreasing
$(-1, 2)$

(15) $y = \frac{2}{3}x^2 - 2x + 7$

yint
(0,7)

Vertex

$x = \frac{2}{2(\frac{2}{3})} = \frac{2}{\frac{4}{3}} = 1.5$

$y = 5.5$

(1.5, 5.5)

(0,7) (1.5, 5.5)

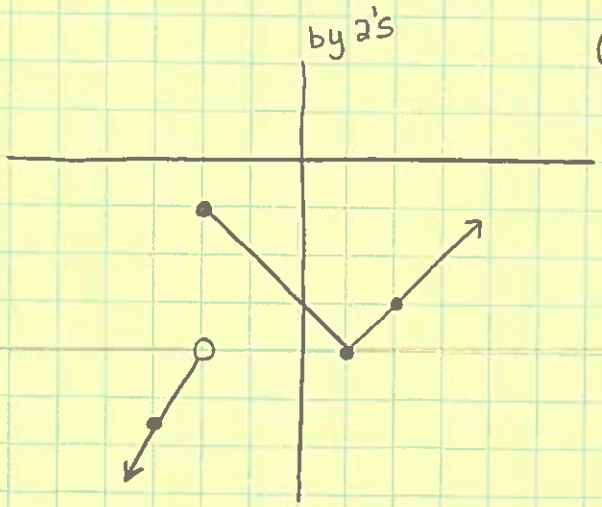
$m = \frac{-1.5}{1.5} = -1$

$y = -x + 7$

(13) a) D: $(-\infty, \infty)$
Not Continuous
Not 1-1

$3x-2$
$(-2,-8)$ ○
$(-3,-11)$

$2|x-1|-8$
$(-2,-2)$ ●
$(1,-8)$
$(2,-6)$

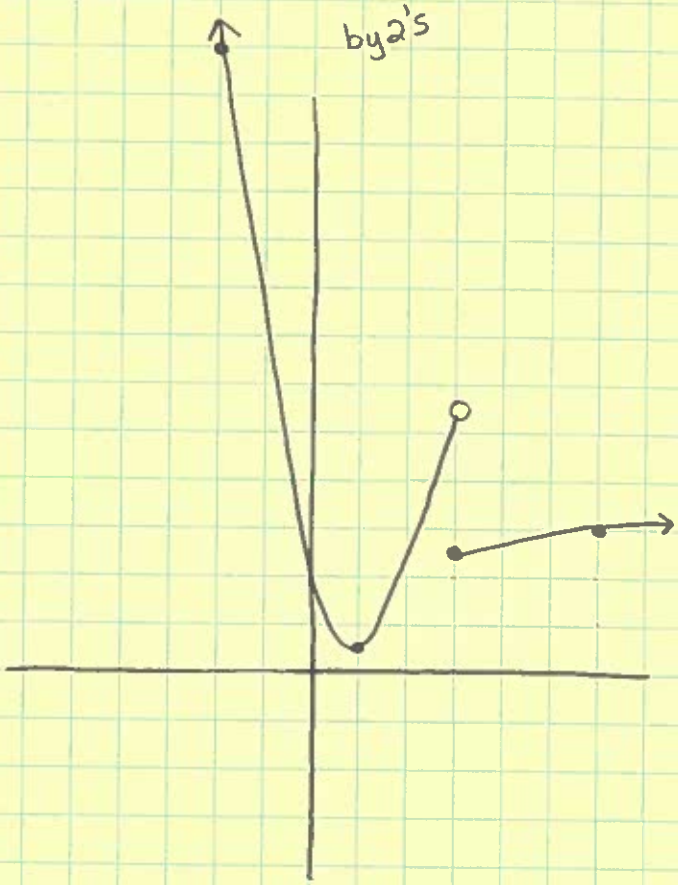


b) D: $(-\infty, \infty)$
Not Continuous
Not 1-1

$3x^2-6x+2$
$(3,11)$ ○
$(1,-1)$
$(-2,26)$

Vertex
$x = \frac{6}{2(3)} = 1$
$y = -1$

$\sqrt{x-2}+4$
$(3,5)$ ●
$(6,6)$



(16) $f(x) = \frac{1}{2}x^2 + 9x - 8$

y int
$(0, -8)$

$(-9, -48.5)$ $(0, -8)$

$m = \frac{40.5}{9} = 4.5$

$y = \frac{9}{2}x - 8$

Vertex

$x = \frac{-9}{2(\frac{1}{2})} = -9$

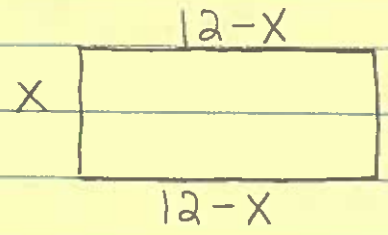
$y = -48.5$

$(-9, -48.5)$

(17) Maximum at $Y = 20.75$

$a < 0$

(18)



$A(x) = x(12 - x)$

6 x 6

maximum

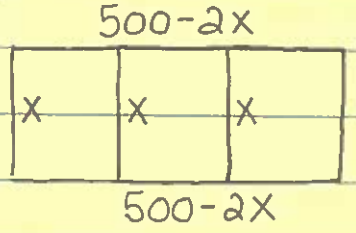
Find Vertex

$(6, 36)$

(19) Find Vertex
$(22.5, 8100)$

22.5 seconds to reach maximum height of 8100 ft

(20)

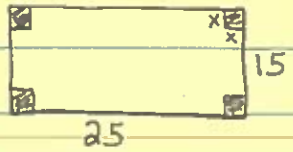


$A(x) = x(500 - 2x)$

Find Vertex (Maximum)

$(125, 31250)$

125 x 250

(21) 

a) $V(x) = l \cdot w \cdot h$

$V(x) = (25-2x)(15-2x)(x)$

$(3.03, 513.05)$

Max Volume $513.05 in^3$

b) $415 = (25-2x)(15-2x)(x)$ Find x-intercepts

$0 = x(25-2x)(15-2x) - 415$

$x = 1.62, 4.66$, ~~13.72~~

(22) Find Maximum point. $(166.19, 3276.23)$

↑ Can't have part of a blender, so round up.

167 blenders to reach maximum profit of $3276.23

(23) Zeros: $3, 2i, -2i, -1$

factors: $(x-3)(x-2i)(x+2i)(x+1)$

$(x-3)(x^2+4)(x+1)$

$\cdot (x^3+4x-3x^2-12)(x+1)$

$(x^3-3x^2+4x-12)(x+1)$

	x^3	$-3x^2$	$4x$	-12
x	x^4	$-3x^3$	$4x^2$	$-12x$
1	x^3	$-3x^2$	$4x$	-12

$y = x^4 - 2x^3 + x^2 - 8x - 12$

(24) $y = 2x^3 - 8x^2 + 9x - 9$

$x = 3$

$$\begin{array}{r|cccc} 3 & 2 & -8 & 9 & -9 \\ & \downarrow & 6 & -6 & 9 \\ \hline & 2 & -2 & 3 & 0 \end{array}$$

$2x^2 - 2x + 3 = 0$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{2 \pm \sqrt{-20}}{4}$$

$$x = \frac{2 \pm 2i\sqrt{5}}{4}$$

$$\boxed{x = \frac{1 \pm i\sqrt{5}}{2}, 3}$$

(25) $y = x^4 - 6x^3 + 13x^2 - 24x + 36$

$x = 3$

$$\begin{array}{r|ccccc} 3 & 1 & -6 & 13 & -24 & 36 \\ & \downarrow & 3 & -9 & 12 & -36 \\ \hline & 1 & -3 & 4 & -12 & 0 \end{array}$$

$x^2 + 4 = 0$ $\quad x - 3 = 0$

$x^2 = -4$ $\quad x = 3$

$x = \pm 2i$

$x^3 - 3x^2 + 4x - 12$

$x^2(x-3) + 4(x-3)$

$(x^2+4)(x-3) = 0$

$\boxed{x = 3_{mq}, \pm 2i}$

(26) a) $(9 - \sqrt{-4})(\sqrt{-9} + \sqrt{-8})$

$(9 - 2i)(3i + 2i\sqrt{2})$

$27i + 18i\sqrt{2} - 6i^2 - 4i^2\sqrt{2}$

$27i + 18i\sqrt{2} + 6 + 4\sqrt{2}$

$\boxed{6 + 4\sqrt{2} + 27i + 18i\sqrt{2}}$

(26) b) $\frac{7i}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{14i-21i^2}{4-9i^2} = \boxed{\frac{21+14i}{13}}$

c) $\frac{5-i}{2+i} \cdot \frac{2-i}{2-i} = \frac{10-7i+i^2}{4-i^2} = \boxed{\frac{9-7i}{5}}$

d) $i^{93} = (i^2)^{46} \cdot i = (-1)^{46} \cdot i = 1 \cdot i = \boxed{i}$

e) $\frac{5x^2+10x}{x^2-x-6} \div \frac{15x^3+45x^2}{x^2-9}$

$\frac{5x(x+2)}{(x-3)(x+2)} \cdot \frac{(x+3)(x-3)}{15x^2(x+3)} = \boxed{\frac{1}{3x} \quad D: x \neq \pm 3, -2, 0}$

f) $\frac{2x^2+x-15}{2x^2-11x-21} \cdot \frac{6x+9}{1} \div \frac{2x-5}{3x-21}$

$6 \times -5 = -30 \quad \frac{2}{6} = \frac{1}{3}$

$6 + -5 = 1 \quad \frac{2}{-5}$

$\frac{(x+3)(2x-5)}{(x-7)(2x+3)} \cdot \frac{3(2x+3)}{1} \cdot \frac{3(x-7)}{2x-5}$

$-14 \times 3 = -42 \quad \frac{2}{-14} = \frac{1}{-7}$

$-14 + 3 = -11 \quad \frac{2}{3}$

$\boxed{9(x+3) \qquad D: x \neq \frac{5}{2}, 7, -\frac{3}{2}}$

g) $\frac{x^2+x-3}{x^2-12x+32} + \frac{3x}{x-8} = \frac{x^2+x-3}{(x-8)(x-4)} + \frac{3x}{x-8}$

$-12 \times \frac{1}{1} = -12$

$\frac{-12}{1} + \frac{1}{1} = -11$

$\frac{4}{-12} = \frac{1}{-3} \quad \frac{4}{1}$

$\frac{x^2+x-3}{(x-8)(x-4)} + \frac{3x(x-4)}{(x-8)(x-4)} = \frac{4x^2-11x-3}{(x-8)(x-4)} = \boxed{\frac{(x-3)(4x+1)}{(x-8)(x-4)} \quad D: x \neq 4, 8}$

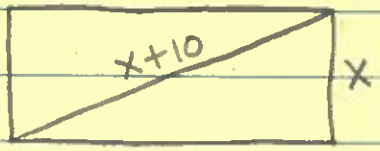
(26) h) $\dfrac{5x-1}{x^2+2x-8} - \dfrac{6}{x+4}$ CD: $(x+4)(x-2)$

$\dfrac{5x-1}{(x+4)(x-2)} - \dfrac{6}{x+4}$

$\dfrac{5x-1}{(x+4)(x-2)} - \dfrac{6(x-2)}{(x+4)(x-2)} = \dfrac{-x+11}{(x+4)(x-2)}$

$= \boxed{\dfrac{-(x-11)}{(x+4)(x-2)} \quad D: x \neq -4, 2}$

(27)



$\sqrt{(x+10)^2 - x^2}$

↑ use pythagorean Theorem

$A(x) = l \cdot w$

$5000 = (\sqrt{(x+10)^2 - x^2})(x)$

$0 = x\sqrt{(x+10)^2 - x^2} - 5000$

$x = 106.08 \approx 106$

Dimensions: $l = \sqrt{(106+10)^2 - 106^2} = 47$

$w = 106$

$\boxed{47 \times 106}$

(28) Zeros: $-2, -2, 0, 2+5i, 2-5i$

Factors: $(x+2)^2(x)(x-2-5i)(x-2+5i)$

$x(x^2+4x+4)(x^2-4x+29)$

$(x^3+4x^2+4x)(x^2-4x+29)$

	x^3	$4x^2$	$4x$
x^2	x^5	$4x^4$	$4x^3$
$-4x$	$-4x^4$	$-16x^3$	$-16x^2$
29	$29x^3$	$116x^2$	$116x$

$\boxed{y = x^5 + 17x^3 + 100x^2 + 116x}$

	x	-2	$5i$
x	x^2	$-2x$	$5xi$
-2	$-2x$	4	$-10i$
$-5i$	$-5xi$	$10i$	$-25i^2$

(29) a) $f(x) = -x^2(2x+3)(x-1)$

EB: Even LC: −

$x \to +\infty, y \to -\infty$

$x \to -\infty, y \to -\infty$

x-int

$x = 0$ (bounce) $x = -\frac{3}{2}$ (linear) $x = 1$ (linear)

3 distinct (different) zeros

b) $P(x) = x^3(2x-1)(x-2)^2$

EB: Even LC: +

$x \to +\infty, y \to +\infty$

$x \to -\infty, y \to +\infty$

x-int

$x = 0$ (POI) $x = \frac{1}{2}$ (linear) $x = 2$ (bounce)

3 distinct zeros

(30) a) ODD LC +

$x \to +\infty, y \to +\infty$

$x \to -\infty, y \to -\infty$

Degree: 5

b) Even LC +

$x \to +\infty, y \to +\infty$

$x \to -\infty, y \to +\infty$

Degree: 4

c) ODD LC −

$x \to +\infty, y \to -\infty$

$x \to -\infty, y \to +\infty$

Degree: 5

d) Even LC +

$x \to +\infty, y \to +\infty$

$x \to -\infty, y \to +\infty$

Degree: 6