

Name: _____

Date: _____

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Q2T2: Trigonometry Study Guide

Honors PreCalculus

ALL WORK IS TO BE DONE ON YOUR OWN PAPER

Remember that this is a study *GUIDE* and not the only material you should study. Studying only the problems that appear on this review guide will not be sufficient. You should also review problems from class starters, notes, and homework assignments for additional practice.

THE UNIT CIRCLE

- To find coterminal angles, add/subtract 360° or 2π
- To convert degrees to radians, multiply angle by $\frac{\pi}{180}$; to convert radians to degrees, $\frac{180}{\pi}$
- Remember where each trigonometric function is positive: "All Students Take Calculus"

1. Find the radian measure of the angles below with the given degree measure.

(a) $195^\circ \cdot \frac{\pi}{180}$

$$\boxed{\frac{13\pi}{12}}$$

(b) $-50^\circ \cdot \frac{\pi}{180}$

$$\boxed{-\frac{5\pi}{18}}$$

(c) $375^\circ \cdot \frac{\pi}{180}$

$$\boxed{\frac{25\pi}{12}}$$

2. Find the degree measure of the angles below with the given radian measure.

(a) $\frac{7\pi}{9} \cdot \frac{180}{\pi}$

$$\boxed{140^\circ}$$

(b) $-\frac{17\pi}{6} \cdot \frac{180}{\pi}$

$$\boxed{-510^\circ}$$

(c) $\frac{13\pi}{8} \cdot \frac{180}{\pi}$

$$\boxed{292.5^\circ}$$

3. Find the angle between 0 and 2π that is coterminal with an angle that measures $\frac{17\pi}{6}$.

$$\frac{17\pi}{6} - \frac{12\pi}{6} = \boxed{\frac{5\pi}{6}}$$

4. Find two angles, one positive and one negative, that are coterminal with the angles given.

(a) -56°

$$\begin{aligned} -56 + 360 &= \boxed{304^\circ} \\ -56 - 360 &= \boxed{-416^\circ} \end{aligned}$$

(b) $\frac{3\pi}{4}$

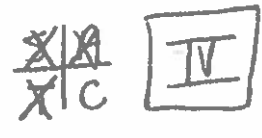
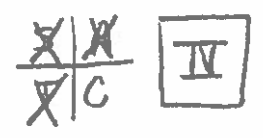
$$\begin{aligned} \frac{3\pi}{4} + \frac{8\pi}{4} &= \boxed{\frac{11\pi}{4}} \\ \frac{3\pi}{4} - \frac{8\pi}{4} &= \boxed{-\frac{5\pi}{4}} \end{aligned}$$

(c) $\frac{5\pi}{8}$

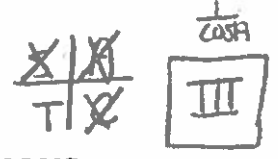
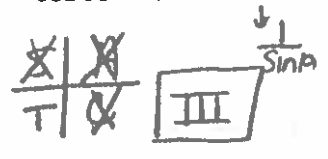
$$\begin{aligned} \frac{5\pi}{8} + \frac{16\pi}{8} &= \boxed{\frac{21\pi}{8}} \\ \frac{5\pi}{8} - \frac{16\pi}{8} &= \boxed{-\frac{11\pi}{8}} \end{aligned}$$

5. In which quadrant are the following conditions met? $\frac{1}{\sin A}$

- (a) $\sin A < 0$ and $\cos A > 0$ (b) $\csc A < 0$ and $\tan A < 0$



- (c) $\cos A < 0$ and $\csc A < 0$ (d) $\cot A > 0$ and $\sec A < 0$



EVALUATING TRIGONOMETRIC FUNCTIONS

- Use unit circle to evaluate trigonometric functions. Remember to determine quadrant where angle is located to determine if the value will be positive or negative. Find coterminal angles on the unit circle when necessary.
- If angle is not "special," use calculator. On calculator, use $\csc = \frac{1}{\sin}$, $\sec = \frac{1}{\cos}$, and $\cot = \frac{1}{\tan}$.

9. $\tan\left(\frac{5\pi}{4}\right) = \frac{S}{T} = \frac{A}{C} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

10. $\cos\left(-\frac{7\pi}{6}\right) = \frac{S}{T} = \frac{A}{C} = \frac{0}{-\frac{\sqrt{3}}{2}} = 0$

11. $\sin(-390^\circ) = \sin 330^\circ = \frac{S}{T} = \frac{A}{C} = \frac{0}{\frac{1}{2}} = 0$

12. $\sin\left(\frac{15\pi}{4}\right) = \sin \frac{7\pi}{4} = \frac{S}{T} = \frac{A}{C} = \frac{0}{-\frac{\sqrt{2}}{2}} = 0$

13. $\tan(60^\circ) = \frac{S}{T} = \frac{A}{C} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

14. $\csc(123^\circ) = \frac{1}{\sin 123^\circ} = 1.192$ (calculator degree mode)

15. $\sec\left(\frac{4\pi}{3}\right) = \frac{S}{T} = \frac{A}{C} = \frac{0}{-\frac{1}{2}} = 0$

16. $\cot(-3\pi) = \cot \pi = \frac{S}{T} = \frac{A}{C} = \frac{0}{0} = \text{und}$

17. $\tan(585^\circ) = \tan 225^\circ = \frac{S}{T} = \frac{A}{C} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

18. $\sec\left(\frac{23\pi}{6}\right) = \sec \frac{11\pi}{6} = \frac{S}{T} = \frac{A}{C} = \frac{0}{\frac{1}{2}} = 0$

19. $\csc(450^\circ) = \csc 90^\circ = \frac{1}{(0,1)} = 1$

20. $\sin\left(\frac{17\pi}{6}\right) = \sin \frac{5\pi}{6} = \frac{S}{T} = \frac{A}{C} = \frac{1}{2}$

21. $\cot\left(\frac{5\pi}{6}\right) = \frac{S}{T} = \frac{A}{C} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

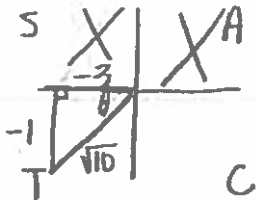
22. $\tan(150^\circ) = \frac{S}{T} = \frac{A}{C} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

23. $\tan\left(\frac{7\pi}{9}\right)$ (calculator radian mode) = -0.839

RIGHT TRIANGLE TRIGONOMETRY

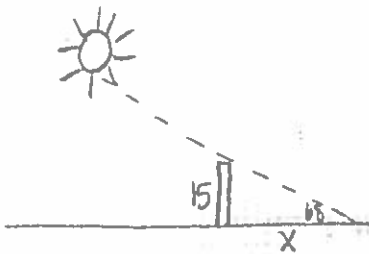
- If the triangle is a RIGHT triangle, use the ratios for right triangles: SOHCAHTOA

24. Given $\tan \theta = \frac{1}{3}$ and $\sin \theta < 0$, find the exact value of all six trigonometric functions.



$$\begin{aligned} \sin \theta &= \frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10} & \csc \theta &= -\sqrt{10} \\ \cos \theta &= \frac{-3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10} & \sec \theta &= -\frac{\sqrt{10}}{3} \\ \tan \theta &= \frac{1}{3} & \cot \theta &= 3 \end{aligned}$$

25. A post is 15 ft tall. How long is the shadow that it casts when the angle of elevation of the sun is 68° ?



$$x \cdot \tan 68 = \frac{15}{x} \cdot x$$

$$x \tan 68 = 15$$

$$x = \frac{15}{\tan 68}$$

$$x = 6.06 \text{ ft}$$

degree mode!

26. Find the perimeter of the triangle below.

degree mode!

$$\cos 60 = \frac{x}{12}$$

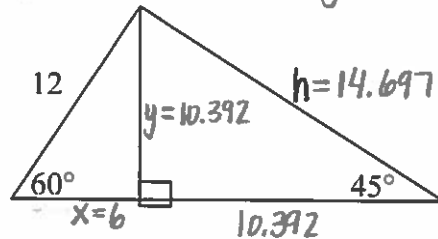
$$12 \cos 60 = x$$

$$6 = x$$

$$\sin 60 = \frac{y}{12}$$

$$12 \sin 60 = y$$

$$10.392 = y$$



$$P = 12 + 14.697 + 10.392 + 6$$

$$P = 43.089$$

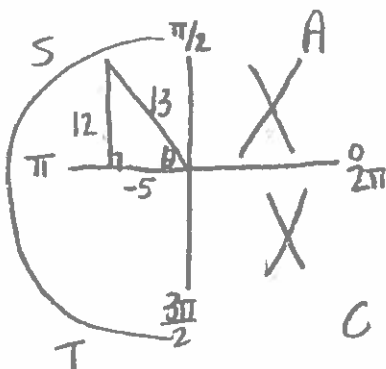
$$\sin 45 = \frac{10.392}{h}$$

$$h \sin 45 = 10.392$$

$$h = \frac{10.392}{\sin 45}$$

$$h = 14.697$$

27. Given $\sin \theta = \frac{12}{13}$ and $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$, find the exact value of all six trigonometric functions.



$$\sin \theta = \frac{12}{13}$$

$$\csc \theta = \frac{13}{12}$$

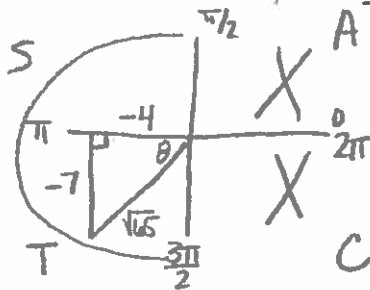
$$\cos \theta = -\frac{5}{13}$$

$$\sec \theta = -\frac{13}{5}$$

$$\tan \theta = -\frac{12}{5}$$

$$\cot \theta = -\frac{5}{12}$$

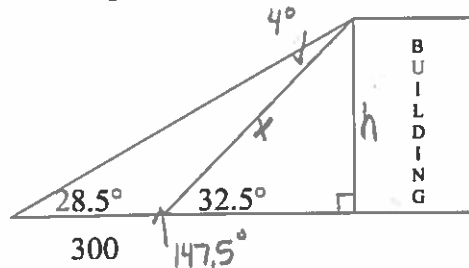
28. Given $\cot \theta = \frac{4}{7}$ and $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$, find the exact value of all six trigonometric functions.



$$\begin{aligned} \sin \theta &= \frac{-7}{\sqrt{65}} = -\frac{7\sqrt{65}}{65} & \csc \theta &= -\frac{\sqrt{65}}{7} \\ \cos \theta &= \frac{-4}{\sqrt{65}} = -\frac{4\sqrt{65}}{65} & \sec \theta &= -\frac{\sqrt{65}}{4} \\ \tan \theta &= \frac{7}{4} & \cot \theta &= \frac{4}{7} \end{aligned}$$

29. To estimate the height of a building above a level plain, a surveyor measures the angle of elevation to the top of the building to be 28.5° . Three hundred feet closer to the building, the angle of elevation is 32.5° . Estimate the height of the building. (Use Law of Sines to start?)

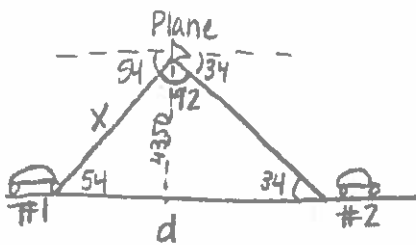
$$\begin{aligned} \frac{\sin 4^\circ}{300} &= \frac{\sin 28.5^\circ}{x} \\ x \sin 4^\circ &= \frac{300 \sin 28.5^\circ}{\sin 4^\circ} \\ x &= 2052.105 \end{aligned}$$



$$\begin{aligned} \sin 32.5^\circ &= \frac{h}{2052.105} \\ 2052.105 \sin 32.5^\circ &= h \\ \boxed{1102.595 \text{ ft} = h} \end{aligned}$$

30. A plane is flying 4,350 ft over a straight road. Two cars are driving along the road on either side of the plane. The angle of depression from the plane to car #1 is 54° and the angle of depression to car #2 is 34° .

- (a) How far is the plane from car #1?
 (b) What is the distance between the two cars? (You may use either right triangles or the Law of Sines to solve this problem.)

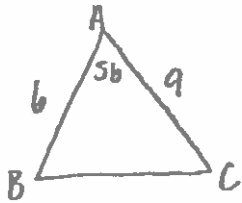


$$\begin{aligned} \text{a) } \sin 54^\circ &= \frac{4350}{x} \\ x \sin 54^\circ &= 4350 \\ x &= \frac{4350}{\sin 54^\circ} \\ \boxed{x = 5376.896 \text{ ft}} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{\sin 34^\circ}{5376.896} &= \frac{\sin 92^\circ}{d} \\ \frac{d \sin 34^\circ}{5376.896} &= \frac{\sin 92^\circ}{\sin 34^\circ} \\ \boxed{d = 9609.601 \text{ ft}} \end{aligned}$$

LAW OF SINES AND LAW OF COSINES * Degree Mode *

31. Solve the triangle that has side lengths of 6 and 9 with an included angle measure of 56° .



$$a^2 = 6^2 + 9^2 - 2(6)(9)\cos 56$$

$$a^2 = 56.667$$

$$a = 7.524$$

$$\frac{\sin 56}{7.524} = \frac{\sin B}{9}$$

$$9 \sin 56 = 7.524 \sin B$$

$$.992 = \sin B$$

$$\sin^{-1}(.992) = B = 82.6^\circ$$

$$\angle A = 56^\circ$$

$$a = 7.524$$

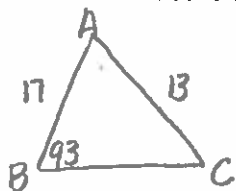
$$\angle B = 82.6^\circ$$

$$b = 9$$

$$\angle C = 41.4^\circ$$

$$c = 6$$

32. Solve the triangle that satisfies $b = 13$, $c = 17$, and $B = 93^\circ$.



(Ambiguous Case)

$$\frac{\sin 93}{13} = \frac{\sin C}{17}$$

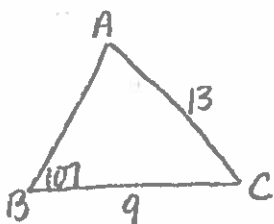
$$13 \sin C = 17 \sin 93$$

$$\sin C = 1.306$$

$$\sin^{-1}(1.306) = C$$

No Triangles exist with these conditions.

33. Solve the triangle that satisfies $a = 9$, $b = 13$, and $B = 107^\circ$.



$$\frac{\sin 107}{13} = \frac{\sin A}{9}$$

$$13 \sin A = 9 \sin 107$$

$$\sin A = .662$$

$$\sin^{-1}(.662) = A$$

$$41.457^\circ = A$$

$$180 - 41.457 = 138.543^\circ = A$$

$\angle A = 41.457^\circ$	$a = 9$
$\angle B = 107^\circ$	$b = 13$
$\angle C = 31.543^\circ$	$c = 7.112$

$$\frac{\sin 107}{13} = \frac{\sin 31.543}{c}$$

$$13 \sin 31.543 = c \sin 107$$

$$7.112 = c$$

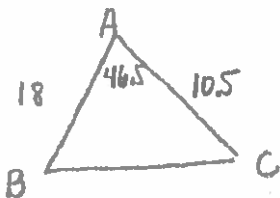
~~$$\angle A = 138.543^\circ$$

$$\angle B = 107^\circ$$

$$\angle C =$$~~

Ambiguous case? No, because $\angle B$ is obtuse (only one triangle works)

34. Solve the triangle that satisfies $A = 46.5^\circ$, $b = 10.5$, and $c = 18.0$.



$$a^2 = 18^2 + 10.5^2 - 2(18)(10.5)\cos 46.5$$

$$a^2 = 174.052$$

$$a = 13.193$$

$$\frac{\sin 46.5}{13.193} = \frac{\sin B}{10.5}$$

$$10.5 \sin 46.5 = 13.193 \sin B$$

$$.577 = \sin B$$

$$\sin^{-1}(.577) = B$$

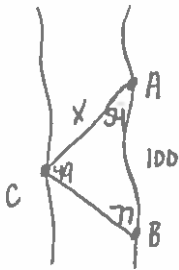
$$35.261$$

$$\angle A = 46.5^\circ \quad a = 13.193$$

$$\angle B = 35.261^\circ \quad b = 10.5$$

$$\angle C = 98.239^\circ \quad c = 18$$

35. To find the distance across a river a surveyor chooses points A and B, 100 m apart and a reference point C on the other side of the river. Approximate the distance between points A and C, given that angle A = 54° and angle B = 77°.

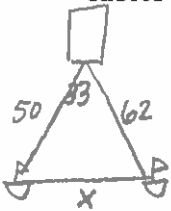


$$\frac{\sin 49}{100} = \frac{\sin 77}{x}$$

$$x \sin 49 = 100 \sin 77$$

$$x = 129.105 \text{ m}$$

36. Two tugboats are pulling a barge with cables 50 m and 62 m long. If the angle between the cables is 33°, approximately how far apart are the tugboats?

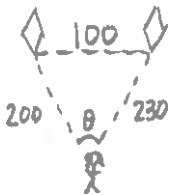


$$x^2 = 50^2 + 62^2 - 2(50)(62)\cos 33$$

$$x^2 = 1144.242$$

$$x = 33.827 \text{ m}$$

37. A girl is flying two kites at the same time. If the strings are 200 ft and 230 ft and the kites are 100 ft apart, what angle do the strings make in her hands?



$$100^2 = 200^2 + 230^2 - 2(200)(230)\cos \theta$$

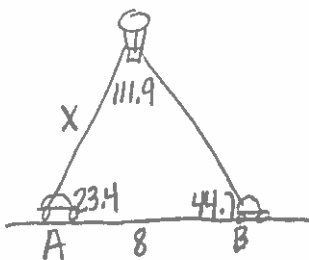
$$-82900 = -92000 \cos \theta$$

$$.901 = \cos \theta$$

$$\cos^{-1}(.901) = \theta$$

$$25.699^\circ = \theta$$

38. A hot air balloon passes over two cars, A and B, 8 miles apart. When the hot air balloon is between the two cars, the angles of elevation from the cars are measured to be 23.4° and 44.7° respectively. Find the distance between the balloon and car A.

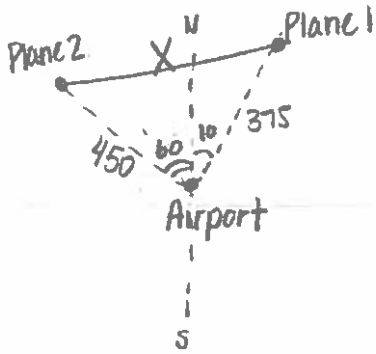


$$\frac{\sin 111.9}{8} = \frac{\sin 44.7}{x}$$

$$x \sin 111.9 = 8 \sin 44.7$$

$$x = 6.065 \text{ miles}$$

39. A plane leaves the airport at 4PM and heads in the direction of 10° east of north at a constant speed of 250 mph. Thirty minutes later, a jet leaves the same airport and flies 60° west of north at a constant speed of 450 mph. Find the distance between the plane and the jet at 5:30PM.



Plane 1: 250 mph for 1.5 hours \rightarrow 375 miles

Plane 2: 450 mph for 1 hour \rightarrow 450 miles

$$X^2 = 450^2 + 375^2 - 2(450)(375)\cos 70$$

$$X^2 = 227693.2016$$

$$X = 477.172 \text{ miles}$$

