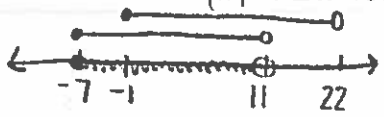
Name: _____ Date: _____ Block: _____

Q1T1: Study Guide **Honors PreCalculus**

ALL WORK IS TO BE DONE ON YOUR OWN PAPER

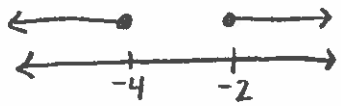
Remember that this is a study GUIDE and not the only material you should study. Studying only the problems that appear on this review guide will not be sufficient. You should also review problems from class starters, notes, and homework assignments for additional practice.

1. Find $A \cap B$ if $A = \{x \mid -7 \le x < 11\}$ and $B = \{x \mid -1 \le x < 22\}$. Write your answer in interval notation.



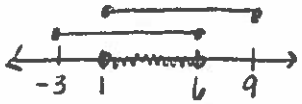
$A \cap B =$ [-1, 11)

2. Find $M \cap N$ if $M = (-\infty, -4]$ and $N = [-2, \infty)$. Write your answer in interval notation.



∅ No intersection

3. Find $[-3,6] \cap [1,9]$. Write your answer in interval notation.

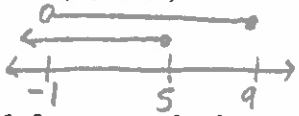


[1, 6] {x | 1 ≤ x ≤ 6}

4. Given $A = \{x \mid -3 < x \le 4\}$ and $B = \{x \mid -1 < x < 7\}$, find $A \cap B$. Write your answer in both set builder notation and interval notation.

$A \cap B = (-1, 4]$; $\{x \mid -1 < x \le 4\}$

5. Given $A = \{x \mid x \le 5\}$ and $B = \{x \mid -1 < x \le 9\}$, find $A \cup B$. Write your answer in interval notation.



$A \cup B = (-\infty, 9]$, $\{x \mid x \le 9\}$

For #6 – 21, factor completely.

6. $(z^2+1)^2 - 7(z^2+1) + 10$

let $a = z^2+1$

$a^2 - 7a + 10$

$(a-5)(a-2)$

$(z^2+1-5)(z^2+1-2)$

$(z^2-4)(z^2-1) \rightarrow (z+2)(z-2)(z+1)(z-1)$

7. $4x(2x+1)^3(5x-4)^2 + 10(5x-4)x^2(2x+1)^4$

$2x(2x+1)^3(5x-4)[2(5x-4) + 5x(2x+1)]$

$2x(2x+1)^3(5x-4)[10x - 8 + 10x^2 + 5x]$

$2x(2x+1)^3(5x-4)[10x^2 + 15x - 6]$

8. $3x^{-1/2} + 4x^{1/2} + x^{3/2}$

$x^{-1/2}(3 + 4x + x^2)$

$x^{-1/2}(x^2 + 4x + 3)$

$x^{-1/2}(x+1)(x+3)$

9. $2x^2 - 5xy + 2y^2$

$(x-2y)(2x-y)$

$\frac{-4}{-4} x \frac{-1}{-1} = 4$

$\frac{-4}{-4} + \frac{-1}{-1} = -5$

10. $(3x^3+5x^2)(-6x-10)$

$x^2(3x+5)-2(3x+5)$

$(3x+5)(x^2-2)$

11. $27x^3-8y^3$

$(3x-2y)(9x^2+6xy+4y^2)$

12. $16-x^4$

$-x^4+16$

$-(x^4-16)$

$-(x^2-4)(x^2+4)$

$-(x-2)(x+2)(x^2+4)$

13. $12t^3-36t^2+27t$

$3t(4t^2-12t+9)$ $-6 \cdot -6 = 36$, $-6+-6=-12$

$3t(2t-3)(2t-3)$

$3t(2t-3)^2$

14. $4x^4-13x^2+9$ $-1 \cdot -4 = 36$, $-9+-4=-13$

$(4x^2-9)(x^2-1)$

$(2x-3)(2x+3)(x-1)(x+1)$

15. $12x^2(4x-1)^3(2x-1)^2+4x(4x-1)^4(2x-1)$

$4x(4x-1)^3(2x-1)[3x(2x-1)+(4x-1)]$

$4x(4x-1)^3(2x-1)[6x^2-3x+4x-1]$

$4x(4x-1)^3(2x-1)[6x^2+x-1]$ $3 \cdot -2 = -6$, $3+-2=1$, $\frac{6x}{3}$ $\frac{6x}{-2}$ $\frac{2x}{1}$ $\frac{3x}{-1}$

$4x(4x-1)^3(2x-1)(2x+1)(3x-1)$

16. x^6+5x^3-24 $8 \cdot -3 = -24$, $8+-3=5$

$(x^3+8)(x^3-3)$

$(x^3-3)(x+2)(x^2-2x+4)$

17. $5x^2-13xy+6y^2$ $-10 \cdot -3 = 30$, $-10+-3=-13$, $\frac{5}{-10}=\frac{1}{-2}$ $\frac{5}{-3}$

$(x-2y)(5x-3y)$

18. $4x^5-9x^3+108x^2-243$

$x^3(4x^2-9)+27(4x^2-9)$

$(4x^2-9)(x^3+27)$

$(2x-3)(2x+3)(x+3)(x^2-3x+9)$

19. $24x^{5/2}-81x^{-1/2}$

$3x^{-1/2}(8x^3-27)$

$3x^{-1/2}(2x-3)(4x^2+6x+9)$

20. $(9x^3 - 27x^2)(-4x + 12)$

$9x^2(x-3) - 4(x-3)$

$(x-3)(9x^2-4)$

$\boxed{(x-3)(3x-2)(3x+2)}$

21. $4x^2(x-1)^2(x+3)^{5/2} - 4x(x-1)^3(x+3)^{3/2}$

$4x(x-1)^2(x+3)^{3/2}[x(x+3) - (x-1)]$

$4x(x-1)^2(x+3)^{3/2}[x^2+3x-x+1]$

$4x(x-1)^2(x+3)^{3/2}[x^2+2x+1]$

$\boxed{4x(x-1)^2(x+3)^{\frac{3}{2}}(x+1)^2}$

22. Find the equation for the line that passes through the points $\left(-\frac{5}{6}, \frac{2}{3}\right)$ and $\left(-\frac{5}{2}, -\frac{4}{3}\right)$.

$m = \frac{-\frac{4}{3} - \frac{2}{3}}{-\frac{5}{2} - \left(-\frac{5}{6}\right)} = \frac{-\frac{6}{3}}{-\frac{15}{6} + \frac{5}{6}} = \frac{-2}{-\frac{10}{6}} = \frac{-2}{-\frac{5}{3}} = -2 \cdot \frac{3}{-5} = \frac{6}{5}$ ← slope

$\boxed{y - \frac{2}{3} = \frac{6}{5}\left(x + \frac{5}{6}\right)}$ OR $y - \frac{2}{3} = \frac{6}{5}x + 1$ $\boxed{y = \frac{6}{5}x + \frac{5}{3}}$ OR $-15\left[\frac{-6}{5}x + y = \frac{5}{3}\right]$ $\boxed{18x - 15y = -25}$

23. Find the equation of the line that passes through the point (-4,5) and is perpendicular to $4x - 6y = -2$.

$\boxed{y - 5 = -\frac{3}{2}(x+4)}$ OR $y - 5 = -\frac{3}{2}x - 6$ $\boxed{y = -\frac{3}{2}x - 1}$ OR $2\left[\frac{3}{2}x + y = -1\right]2$ OR $\boxed{3x + 2y = -2}$

$\frac{-6y}{-6} = \frac{-4x}{-6} - \frac{2}{-6}$

$y = \frac{2}{3}x + \frac{1}{3}$

↓ slope

24. Find the equation of the line that passes through the point (-5,7) and is parallel to the line that passes through the points (-1,4) and (-7,8).

$m = \frac{8-4}{-7-(-1)} = \frac{4}{-6} = -\frac{2}{3}$

$\boxed{y - 7 = -\frac{2}{3}(x+5)}$ OR $y - 7 = -\frac{2}{3}x - \frac{10}{3}$ $\boxed{y = -\frac{2}{3}x + \frac{11}{3}}$ OR $3\left[\frac{2}{3}x + y = \frac{11}{3}\right]$ OR $\boxed{2x + 3y = 11}$

25. Find the equation of the perpendicular bisector to the line segment with endpoints (-4,5) and (-6,8).

$m = \frac{2}{3}$

$MP = \left(\frac{-6+-4}{2}, \frac{8+5}{2}\right)$

$= \left(-5, 13/2\right)$

$m = \frac{8-5}{-6-(-4)} = \frac{3}{-2}$

$\boxed{y - \frac{13}{2} = \frac{2}{3}(x+5)}$ $y - \frac{13}{2} = \frac{2}{3}x + \frac{10}{3}$ $\boxed{y = \frac{2}{3}x + \frac{59}{6}}$ OR $-6\left[-\frac{2}{3}x + y = \frac{59}{6}\right]-6$ OR $\boxed{4x - 6y = -59}$

26. In 1945, when the United Nations was formed, there were 55 members. In 1987, 158 nations were members. Write a linear equation that gives the number of nations N in the UN in terms of the year t. Discuss the significance of the slope in this model. Then use the model to estimate how many nations were in the UN at the beginning of the Vietnam War in 1959.

let 1945 = year 0

(0,55) (42,158)

$m = \frac{158-55}{42-0} = \frac{103}{42} \approx 2.452$

The number of members in the UN is increasing at a rate of 2.452 nations per year.

$N = 2.452t + 55$

In 1959, $t = 14$

$N = 2.452(14) + 55$

$N = 89.333$ Nations

27. For a child between the ages of 6 and 16, its height h (in inches) is approximately a linear function of age t (in years). Joe Smith's son John was 24 inches tall at age 6 and 38 inches tall at age 10. Find a linear model for John's height h as a function of his age t. Discuss the significance of the slope in this model. Predict what John's height will be at age 16. (Since the model only applies from $t = 6$ to $t = 16$, let 6 = year 0)

(6, 24) (10, 38)

$m = \frac{38-24}{10-6} = \frac{14}{4} = 3.5$

John's height is increasing at a rate of 3.5 inches per year.

$y - 24 = 3.5(x - 6)$

$y - 24 = 3.5x - 21$

$y = 3.5x + 3$

age 16, $t = 10$

$h = 3.5(10) + 3$

$h = 59$ inches

28. A high definition television that was purchased for $3,200 completely depreciates in 8 years. Assuming the rate of depreciation is constant, find a linear model and interpret the meaning of the slope.

(0,3200) (8,0)

$m = \frac{0-3200}{8-0} = \frac{-3200}{8} = -400$

The value of the television is decreasing at a rate of $400 each year.

$y = -400x + 3200$

29. A real estate office handles an apartment complex with 50 units. When the rent per month is $580 per month, all 50 units are occupied. However, when the rent is $625 per month, the average number of occupied units drops to 47. Assume that the relationship between the monthly rent, p, and the demand, x, is linear. Find a linear model for the information and interpret the meaning of the slope. Predict the number of units occupied when the rent is $595.

(580, 50) (625, 47)

$m = \frac{47-50}{625-580} = \frac{-3}{45} = -\frac{1}{15} = -.067$

The number of units rented decreases at a rate of .067 units per dollar, or one unit for every $15 increase in rent.

$x - 50 = -\frac{1}{15}(p - 580)$

$x - 50 = -\frac{1}{15}p + \frac{116}{3}$

$x = -\frac{1}{15}p + \frac{266}{3}$ OR $x = -.067p + 88.667$

$p = \$595$

$x = -\frac{1}{15}(595) + \frac{266}{3}$

$x = 49$ apartments

32. Sketch a graph that meets the following requirements.

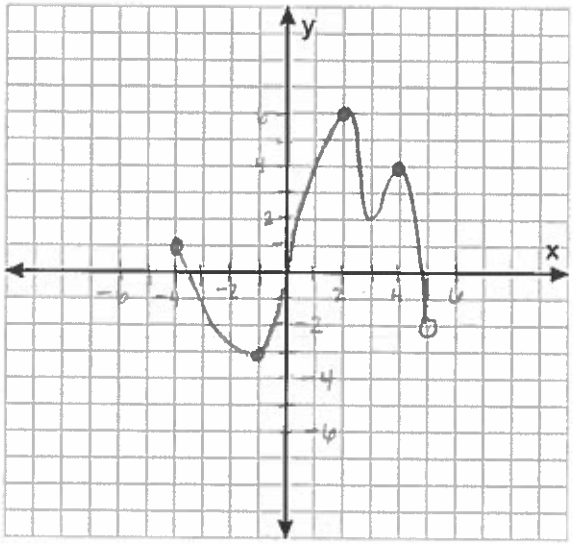
(a) Domain: [-4, 5)

(b) Range: [-3, 6]

(c) Absolute maximum value at $x = 2$

(d) Absolute minimum value at $x = -1$

(e) Relative maximum value $x = 4$



* Answers will vary... This is just a sketch, just make sure the 5 requirements are met.

Be sure to review the Linear Functions and Graph Analysis class example worksheets that we worked in class on the first few days of the course, as well as all class starters.

30. Simplify the following. Be sure to leave no radicals in the denominator.

(a) $\left(8x^3y^{1/2}\right)^{2/3}\left(32x^4y^3\right)^{1/2}$

$\sqrt{32} = 4\sqrt{2}$

$8^{2/3}x^2y^{1/3} \cdot 32^{1/2}x^2y^{3/2}$

$4x^2y^{1/3} \cdot 4\sqrt{2} \cdot x^2y^{3/2}$

$\boxed{16\sqrt{2}\,x^4y^{11/6}}$

(b) $\left(\frac{6a^4b}{4ac^{-2}}\right)^{-1}\left(\frac{8ab^{-3}}{12b^2c^{-5}}\right)^2$

$\frac{4ac^{-2}}{6a^4b} \cdot \frac{64a^2b^{-6}}{144b^4c^{-10}}$

$\frac{256a^3b^{-6}c^{-2}}{864a^4b^5c^{-10}} = \boxed{\frac{8c^8}{27ab^{11}}}$

(c) $\left(\frac{2x^{-1}y^{5/6}}{3z^{-5/3}}\right)^{-3}\left(\frac{8x^4z^{3/4}}{9y^{7/8}}\right)^2$

$\frac{2^{-3}x^3y^{-5/2}}{3^{-3}z^5} \cdot \frac{64x^8z^{3/2}}{81y^{7/4}}$

$\frac{3^3 \cdot 64x^{11}y^{-5/2}z^{3/2}}{2^3 \cdot 81y^{7/4}z^5} = \frac{1728x^{11}}{648y^{17/4}z^{7/2}} = \boxed{\frac{8x^{11}}{3y^{17/4}z^{7/2}}}$

(d) $\sqrt{128a^4b^{11}c}$

$64 \cdot 2$

$\boxed{8a^2b^5\sqrt{2bc}}$

(e) $\sqrt[3]{54x^{10}y^7}$

$27 \cdot 2$

$\boxed{3x^3y^2\sqrt[3]{2xy}}$

(f) $\frac{\sqrt{3}}{\sqrt{6}+\sqrt{x}}$

$\frac{(\sqrt{6}-\sqrt{x})}{(\sqrt{6}-\sqrt{x})}$

$\frac{\sqrt{18}-\sqrt{3x}}{\sqrt{36}-\sqrt{6x}+\sqrt{6x}-\sqrt{x^2}} = \boxed{\frac{3\sqrt{2}-\sqrt{3x}}{6-x}}$

$9 \cdot 2$

31. Rationalize the numerator in the following expressions.

(a) $\frac{\sqrt{x+36}+6}{x}$, $\frac{\sqrt{x+36}-6}{\sqrt{x+36}-6}$

$\frac{x+36-36}{x(\sqrt{x+36}-6)} = \frac{x}{x\sqrt{x+36}-6x}$

$= \boxed{\frac{1}{\sqrt{x+36}-6}}$

(b) $\frac{\sqrt{x^2+16}-5}{x-3} \cdot \frac{\sqrt{x^2+16}+5}{\sqrt{x^2+16}+5}$

$\frac{x^2+16-25}{(x-3)(\sqrt{x^2+16}+5)} = \frac{x^2-9}{(x-3)(\sqrt{x^2+16}+5)}$

$\frac{(x-3)(x+3)}{(x-3)(\sqrt{x^2+16}+5)} = \boxed{\frac{x+3}{\sqrt{x^2+16}+5}}$