Name: _____ Date: _____ Block: ____

Q1T3: Study Guide **Honors PreCalculus**

ALL WORK IS TO BE DONE ON YOUR OWN PAPER

Remember that this is a study GUIDE and not the only material you should study. Studying only the problems that appear on this review guide will not be sufficient. You should also review problems from class starters, notes, and homework assignments for additional practice.

Dividing Polynomials

1. Find the quotient and remainder using synthetic division.

(a) $\dfrac{9x^3-6x^2+3x-4}{x-\frac{1}{3}}$

$\frac{1}{3}$ | 9 -6 3 -4
↓ 3 -1 2/3
9 -3 2 | 10/3

$9x^2-3x+2-\dfrac{10/3}{x-\frac{1}{3}}$

(b) $\dfrac{4x^4+10x^3-8x+1}{x-1}$

1 | 4 10 0 -8 1
↓ 4 14 14 6
4 14 14 6 | 7

$4x^3+14x^2+14x+6+\dfrac{7}{x-1}$

2. Use the remainder theorem to find the value of *P(-2)*: $P(x)=x^4-5x^2+1$

$P(-2)=(-2)^4-5(-2)^2+1=-3$ remainder $=-3$

3. Use the remainder theorem to find the value of *P(4)*: $P(x)=x^5-9x^2+11x+5$

$P(4)=(4)^5-9(4)^2+11(4)+5=929$ remainder $=929$

4. Show that $x-2$ is a factor of the polynomial $P(x)=x^3+x^2-11x+10$.

2 | 1 1 -11 10
↓ 2 6 -10
1 3 -5 | 0

yes, $x-2$ is a factor because the remainder is 0.

5. Show that $x = -1/3$ is a zero of the polynomial $P(x)=27x^4-9x^3+3x^2+6x+1$.

$-\frac{1}{3}$ | 27 -9 3 6 1
↓ -9 6 -3 -1
27 -18 9 3 | 0

$x=-\frac{1}{3}$ is a zero because the remainder is 0.

6. Show that $x-2i$ is a factor of the polynomial $P(x)=4x^4-15x^3+25x^2-60x+36$.

2i | 4 -15 25 -60 36
↓ 8i -16-30i 60+18i -36
4 -15+8i 9-30i 18i | 0

$x-2i$ is a factor because the remainder is 0.

7. What is the remainder if $P(x) = 3x^{100} + 5x^{85} - 4x^{38} + 2x^{17} - 6$ is divided by $x + 1$?

$P(-1) = 3(-1)^{100} + 5(-1)^{85} - 4(-1)^{38} + 2(-1)^{17} - 6$

$3 - 5 - 4 - 2 - 6 = -14$ The remainder is -14.

8. Find the zeros of $P(x) = 24x^4 - 206x^3 + 629x^2 - 786x + 315$ if $x = 3$ and $x = ¾$ are two of the zeros of the polynomial.

	24	−206	629	−786	315
3	↓	72	−402	681	−315
	24	−134	227	−105	0
¾	↓	18	−87	105	
	24	−116	140	0	

$24x^2 - 116x + 140$

$4(6x^2 - 29x + 35)$ $\quad -15 \times -14 = 210$

$4(2x-5)(3x-7) = 0$ $\quad -15 + -14 = -29$

$x = \frac{5}{2}, \frac{7}{3}, 3, \frac{3}{4}$ $\quad \frac{6x}{-15} = \frac{2x}{-5}$ $\quad \frac{6x}{-14} = \frac{3x}{-7}$

Complex Numbers

Simplify the following. Write all answers in the form $a + bi$. $i^2 = -1$

9. $3(5 + 8i) - (4 - 2i)(5 + 6i)$ → $32 + 14i$

$15 + 24i - (20 + 24i - 10i - 12i^2)$

$15 + 24i - 32 - 14i$

$-17 + 10i$

10. $7i(5 - 2i) + (8 + 2i)(4 - 3i)$

$35i - 14i^2 + [32 - 24i + 8i - 6i^2]$

$35i + 14 + 38 - 16i$

$52 + 19i$

11. $\frac{2-4i}{5+3i} \cdot \frac{5-3i}{5-3i}$

$\frac{10 - 6i - 20i + 12i^2}{25 - 9i^2}$

$\frac{-2 - 26i}{34} = \frac{-1 - 13i}{17}$

12. $\frac{3i}{2-i} \cdot \frac{2+i}{2+i} = \frac{6i + 3i^2}{4 - 2i + 2i - i^2}$

$= \frac{-3 + 6i}{5}$

13. i^{22}

$(i^2)^{11}$

$(-1)^{11} = -1$

14. i^{39}

$(i^2)^{19} \cdot i$

$(-1)^{19} \cdot i$

$-i$

Zeros of Polynomials

Find the zeros of the following polynomials.

15. $P(x)=x^3+x^2-14x-24$

P: ±1 ±2 ±3 ±4 ±6 ±8 ±12 ±24

q: ±1

P/q: P

4 | 1 1 −14 −24
↓ 4 20 24
1 5 6 10

x^2+5x+6

$(x+3)(x+2)=0$

$x=-3,-2,4$

16. $P(x)=12x^3+8x^2-3x-2$

P: ±1 ±2

q: ±1 ±2 ±3 ±4 ±6 ±12

P/q: ±1 ±½ ±⅓ ±¼ ±⅙ ±1/12 ±2 ±⅔

½ | 12 8 −3 −2
↓ 6 7 2
12 14 4 10

$12x^2+14x+4$ $\frac{4}{4}\times\frac{3}{3}=12$ $\frac{4}{4}+\frac{3}{3}=7$

$2(6x^2+7x+2)=0$ $\frac{6x}{4}=\frac{3x}{2}$ $\frac{6x}{3}=\frac{2x}{1}$

$2(3x+2)(2x+1)=0$

$x=-\frac{2}{3},-\frac{1}{2},\frac{1}{2}$

17. $P(x)=8x^3+18x^2+45x+27$

P: ±1 ±3 ±9 ±27

q: ±1 ±2 ±4 ±8

P/q: ±1 ±½ ±¼ ±⅛ ±3 ±3/2 ±¾ ±⅜ ±27 ±27/2 ±27/4 ±27/8

−¾ | 8 18 45 27
↓ −6 −9 −27
8 12 36 10

$8x^2+12x+36=0$

$4(2x^2+3x+9)=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-3\pm\sqrt{3^2-4(2)(9)}}{2(2)}$

$x=\frac{-3\pm\sqrt{-63}}{4}$ $\frac{9\cdot7}{}$

$x=\frac{-3\pm3i\sqrt{7}}{4},-\frac{3}{4}$

18. $P(x)=x^4+3x^3-30x^2-6x+56$

P: ±1 ±2 ±4 ±7 ±8 ±14 ±28 ±56

q: ±1

P/q = P

4 | 1 3 −30 −6 56
↓ 4 28 −8 −56
−7 | 1 7 −2 −14 10
↓ −7 0 14
1 0 −2 10

$x^2-2=0$

$x^2=\sqrt{2}$

$x=\pm\sqrt{2},-7,4$

19. $P(x)=6x^4+5x^3-17x^2-6x$

$x(6x^3+5x^2-17x-6)$

P: ±1 ±2 ±3 ±6

q: ±1 ±2 ±3 ±6

P/q: ±1 ±½ ±⅓ ±⅙ ±2 ±⅔ ±3 ±3/2 ±6

−2 | 6 5 −17 −6
↓ −12 14 6
6 −7 −3 10

$6x^2-7x-3$ $\frac{-9}{-1}\times\frac{2}{2}=-18$ $-1+2=-7$

$(3x+1)(2x-3)=0$ $\frac{6x}{-9}=\frac{2x}{-3}$ $\frac{6x}{2}=\frac{3x}{1}$

$x=-\frac{1}{3},\frac{3}{2},-2,0$

20. $P(x)=3x^5-10x^4-6x^3+24x^2+11x-6$

P: ±1 ±2 ±3 ±6

q: ±1 ±3

P/q: ±1 ±⅓ ±2 ±⅔ ±3 ±6

−1 | 3 −10 −6 24 11 −6
↓ −3 13 −7 −17 6
3 | 3 −13 7 17 −6 10
↓ 9 −12 −15 6
2 | 3 −4 −5 2 10
↓ 6 4 −2
3 2 −1 10

$3x^2+2x-1=0$ $\frac{3}{3}\times\frac{-1}{1}=-3$ $\frac{3}{3}+\frac{-1}{1}=2$

$(3x-1)(x+1)=0$

$x=\frac{1}{3},-1$ $\frac{3x}{3}=\frac{x}{1}$ $\frac{3x}{-1}$

$x=\frac{1}{3},3,2,-1\ (\text{mult. of } 2)$

Find the polynomial with real coefficients that satisfies the given conditions.

21. Degree = 4; zeros at -1 (multiplicity 3) and 4, as $x \to -\infty\ y \to -\infty$ and $x \to \infty\ y \to -\infty$

$x=-1 \quad x=-1 \quad x=-1 \quad x=4$

$(x+1)(x+1)(x+1)(x-4)$

$(x^2+2x+1)(x^2-3x-4)$

Even → ↕ (negative)

$x^4-3x^3-4x^2+2x^3-6x^2-8x+x^2-3x-4$

$-(x^4-x^3-9x^2-11x-4) = \boxed{-x^4+x^3+9x^2+11x+4}$

22. Degree = 3; zeros at -2 and $4i$; leading coefficient of 4

$x=-2 \quad x=4i \quad x=-4i$

$(x+2)(x-4i)(x+4i) \quad i^2=-1$

$(x+2)(x^2+4ix-4ix-16i^2)$

$(x+2)(x^2+16)$

$x^3+16x+2x^2+32$

$4(x^3+2x^2+16x+32)$

$\boxed{4x^3+8x^2+64x+128}$

23. Degree = 4; zeros at $\sqrt{5}$ and $2+3i$

$x=\sqrt{5} \quad x=-\sqrt{5} \quad x=2+3i \quad x=2-3i$

$(x-\sqrt{5})(x+\sqrt{5})(x-2-3i)(x-2+3i)$

$(x^2-5)(x^2-2x+3ix-2x+4-6i-3ix+6i-9i^2)$ ← $i^2=-1$

$(x^2-5)(x^2-4x+13)$

$x^4-4x^3+13x^2-5x^2+20x-65 \longrightarrow \boxed{x^4-4x^3+8x^2+20x-65}$

24. Degree = 3; zeros at -2 and $3-4i$; constant term is 10 → multiply by $\frac{1}{5}$

$x=-2 \quad x=3-4i \quad x=3+4i$

$(x+2)(x-3+4i)(x-3-4i)$

$(x+2)(x^2-3x-4ix-3x+9+12i+4ix-12i-16i^2)$ ← $i^2=-1$

$(x+2)(x^2-6x+25)$

$x^3-6x^2+25x+2x^2-12x+50$

$\frac{1}{5}(x^3-4x^2+13x+50) \longrightarrow \boxed{\frac{1}{5}x^3-\frac{4}{5}x^2+\frac{13}{5}x+10}$

25. Degree = 5; zeros at -2 (multiplicity of 2), 0, and $4i$

$x=-2 \quad x=-2 \quad x=0 \quad x=4i \quad x=-4i$

$x(x+2)(x+2)(x-4i)(x+4i)$

$x(x^2+4x+4)(x^2-16i^2)$

$(x^3+4x^2+4x)(x^2+16)$

$x^5+16x^3+4x^4+64x^2+4x^3+64x$

$\boxed{x^5+4x^4+20x^3+64x^2+64x}$

$h=8$

26. A canvas camping tent is to be constructed in the shape of a pyramid with a square base. An 8 foot pole will form the center support. Find the length x of a side of the base so that the volume of the tent is 584 ft^3.

The volume for a pyramid with a square base is $V=\frac{1}{3}x^2h$, where x is the length of one side of the base and h is the height of the pyramid.

$584=\frac{1}{3}x^2(8)$

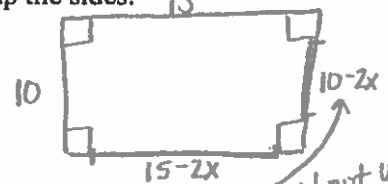
$584=\frac{8}{3}x^2$

$\sqrt{219}=\sqrt{x^2}$

$\pm 14.799=x$

$X=14.799\text{ ft}=\text{side}$

27. From a rectangular piece of cardboard having dimensions 10 inches by 15 inches, an open box is to be made by cutting out identical squares of length x from each corner and turning up the sides.



(a) What value(s) of x gives a volume of 90 in^3?

$V=lwh$ $\quad 90=x(10-2x)(15-2x)$

$90=x(150-20x-30x+4x^2)$

$90=x(4x^2-50x+150)$

$0=4x^3-50x^2+150x-90$

$X=.799, 3.385, 6.316$ would not work here

(b) What is the maximum volume of a box that can be made from the piece of cardboard?

$V=x(10-2x)(15-2x)$

$V=4x^3-50x^2+150x$

(use calculator to find maximum)

change window → y-max

Maximum

(1.962, 132.038)

Cut 1.962 in squares for a max volume of 132.038 in^3

28. It can be shown by means of calculus that the rate R (in deer per year) at which the deer population changes at time t is given by $R=-4t^3+42t$. What is the maximum rate of change in the deer population? When does the rate of change in the deer population equal zero?

Maximum: (1.87, 52.383)

The maximum rate of change in the deer population is 53 deer per year.

(3.24, 0)

The rate of change in deer population is 0 in about 3.24 years.

29. The rabbit population on a small island is observed to be given by the function $P=120t-0.4t^4+1000$ where t is the time measured in months since observations began. When does the population disappear from the island?

(8.42, 0)

The population disappears from the island about 8.42 months since observations began.

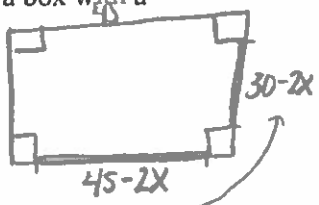
30. A box with an open top is to be constructed from a 30 inch by 45 inch piece of cardboard by cutting squares from each corner and folding up the sides. Find the value(s) of x that would produce a box with a volume of 2700 cubic inches. $V = lwh$ $V = x(30-2x)(45-2x)$

$2700 = x(1350 - 60x - 90x + 4x^2)$

$2700 = 4x^3 - 150x^2 + 1350x$

$0 = 4x^3 - 150x^2 + 1350x - 2700$

$x = 2.814, 9.541, 25.196$ ← too large



Graphing Polynomial Functions

Sketch the graph of the following polynomials. Be sure to identify the x-intercepts, the y-intercept, and the end behavior.

31. $P(x) = (x+1)(x-2)(x-5)$

$x = -1, 2, 5$

y-int: $(1)(-2)(-5)$

$x = 0$ 10

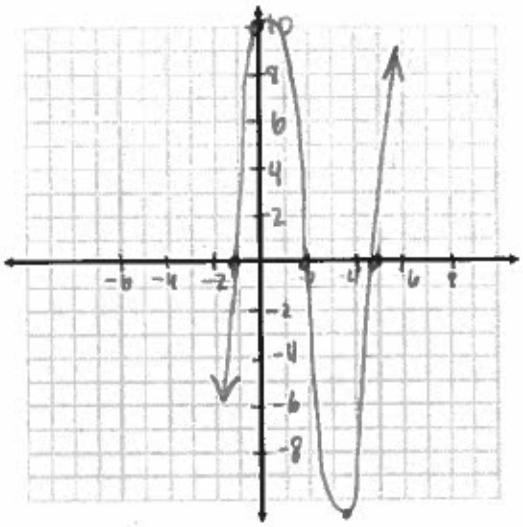
$(0, 10)$

as $x \to \infty$, $y \to \infty$

as $x \to -\infty$, $y \to -\infty$

rel. max: $(.268, 10.39)$

rel. min: $(3.73, -10.39)$



32. $P(x) = x^4 - 17x^2 + 16$

$(x^2 - 16)(x^2 - 1)$

$(x-4)(x+4)(x-1)(x+1)$

$x = 4, -4, 1, -1$

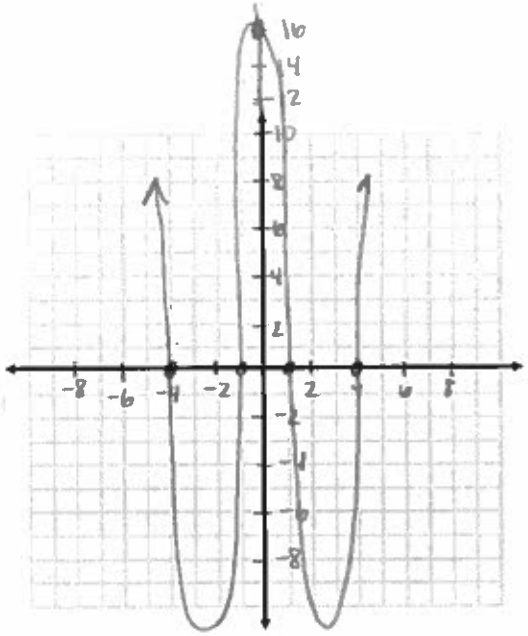
y-int: $(0)^4 - 17(0)^2 + 16$

$x = 0$ 16

$(0, 16)$

as $x \to \infty$, $y \to \infty$

as $x \to -\infty$, $y \to \infty$



* If you find the relative min. values you can see they actually go down to -56.25.

$(-2.92, -56.25)$ and $(2.92, -56.25)$

33. $P(x)=(x^3-3x^2)(-9x+27)$

$x^2(x-3)-9(x-3)$

$(x-3)(x^2-9)$

$(x-3)(x-3)(x+3)=0$

$x=-3,3$ (mult. of 2)

y-int: $(0)^3-3(0)^2-9(0)+27$

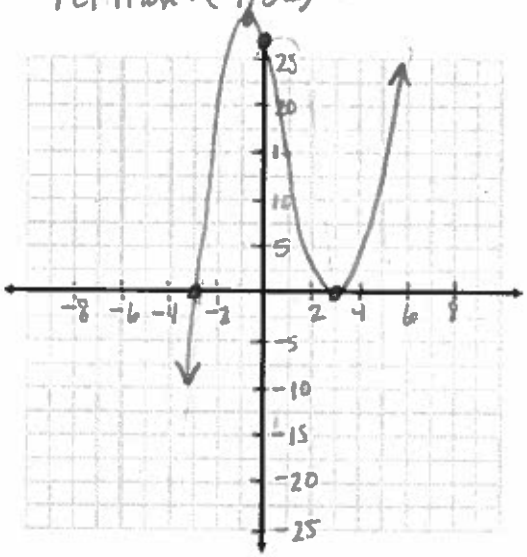
$x=0$ $\quad 27$

$(0,27)$

as $x\to\infty, y\to\infty$

as $x\to-\infty, y\to-\infty$

rel max: $(-1,32)$



34. $P(x)=x^4-x^3-11x^2+9x+18$

$p: \pm1 \pm2 \pm3 \pm6 \pm9 \pm18$

$q: \pm1$

$p/q=p$

-1	1	-1	-11	9	18
	↓	-1	2	9	-18
2	1	-2	-9	18	0
	↓	2	0	-18	
	1	0	-9	0	

y-int: $(0,18)$

$x=0$

as $x\to\infty, y\to\infty$

as $x\to-\infty, y\to\infty$

$x^2-9=0$

$(x-3)(x+3)=0$

$x=3,-3,-1,2$