Name: ______________________________ Block: _____

Date: ___________

Q1T4: Study Guide

Honors PreCalculus

Simplify the following rational expressions. Be sure to state the excluded values.

1) $\frac{a-4}{a^2-16} \times \frac{a^2+5a+4}{a-5}$

$\frac{\cancel{a-4}}{(\cancel{a-4})(\cancel{a+4})} \cdot \frac{(\cancel{a+4})(a+1)}{a-5}$

$\boxed{\frac{a+1}{a-5}, a \neq \pm 4, 5}$

2) $\frac{x^2-15x+54}{x^2-14x+48} \div \frac{1}{x-8}$

$\frac{(x-9)(\cancel{x-6})}{(\cancel{x-6})(\cancel{x-8})} \cdot \frac{(\cancel{x-8})}{1}$

$\boxed{x-9, \ x \neq 6, 8}$

3) $\frac{x-4}{x^2+5x+6} + \frac{x-1}{x^2-4}$

$\frac{x-4}{(x+2)(x+3)} + \frac{x-1}{(x+2)(x-2)}$

$\frac{(x-4)(x-2)}{(x+2)(x-2)(x+3)} + \frac{(x-1)(x+3)}{(x+2)(x-2)(x+3)}$

$\frac{x^2-6x+8+x^2+2x-3}{(x+2)(x-2)(x+3)} = \boxed{\frac{2x^2-4x+5}{(x+2)(x-2)(x+3)}, x \neq \pm 2, -3}$

4) $\frac{x}{x^2-x-6} - \frac{1}{x+2} - \frac{2}{x-3}$

$(x-3)(x+2)$

$\frac{x}{(x-3)(x+2)} - \frac{x-3}{(x-3)(x+2)} - \frac{2(x+2)}{(x-3)(x+2)}$

$\frac{x-(x-3)-2(x+2)}{(x-3)(x+2)}$

$\frac{x-x+3-2x-4}{(x-3)(x+2)} = \frac{-2x-1}{(x-3)(x+2)} = \boxed{\frac{-(2x+1)}{(x-3)(x+2)} \quad x \neq 3, -2}$

5) $\frac{\frac{3}{x}+\frac{5}{x+3}}{\frac{x}{1}}$ CD: $x(x+3)$

$\frac{3(x+3)+5x}{x^2(x+3)} = \frac{3x+9+5x}{x^2(x+3)}$

$\boxed{\frac{8x+9}{x^2(x+3)}, x \neq 0, -3}$

6) $\frac{(6x+h)^{-1}-(6x)^{-1}}{h}$

$\frac{\frac{1}{6x+h} - \frac{1}{6x}}{\frac{h}{1}}$ CD: $6x(6x+h)$

$\frac{6x-(6x+h)}{6xh(6x+h)} = \frac{6x-6x-h}{6xh(6x+h)}$

$= \frac{-h}{6xh(6x+h)} = \boxed{\frac{-1}{6x(6x+h)}, x \neq 0, -\frac{h}{6} \quad h \neq 0, -6x}$

$6x^2(x-1) \quad 3(x-4)(x-1)$

7) $\frac{x}{x-3} - \frac{x+1}{x^2+5x-24}$

$\frac{x}{x-3} - \frac{x+1}{(x+8)(x-3)}$

$\frac{x(x+8)}{(x+8)(x-3)} - \frac{x+1}{(x+8)(x-3)}$

$\frac{x(x+8)-(x+1)}{(x+8)(x-3)} = \frac{x^2+8x-x-1}{(x+8)(x-3)}$

$\boxed{\frac{x^2+7x-1}{(x+8)(x-3)}, x \neq 3, -8}$

8) $\frac{6x^3-6x^2}{x^4+5x^3} \div \frac{3x^2-15x+12}{2x^2+2x-40}$

$x^3(x+5) \quad 2(x+5)(x-4)$

$\frac{6x^2\cancel{(x-1)}}{x^3\cancel{(x+5)}} \cdot \frac{2\cancel{(x+5)}\cancel{(x-4)}}{3\cancel{(x-4)}\cancel{(x-1)}}$

$\frac{12x^2}{3x^3} = \boxed{\frac{4}{x}, x \neq 0, 1, 4, -5}$

9) $\dfrac{\frac{5}{x+3} - \frac{5}{1}}{\frac{6}{1} + \frac{3}{2x+6}}$ CD: $2(x+3)$

$2(x+3)$

$\frac{10 - 10(x+3)}{12(x+3)+3} = \frac{10-10x-30}{12x+36+3}$

$= \frac{-10x-20}{12x+39} = \boxed{\frac{-10(x+2)}{3(4x+13)}, x \neq -\frac{13}{4}, -3}$

10) $\dfrac{\frac{2}{x^2-9}}{\frac{x}{x+3} + \frac{1}{1}}$ CD: $(x+3)(x-3)$

$\frac{2}{x(x-3)+(x+3)(x-3)} = \frac{2}{x^2-3x+x^2-\cancel{3x}+\cancel{3x}-9}$

$= \frac{2}{2x^2-3x-9}$

$-6 \times 3 = -18$

$-6 + 3 = -3$

$\frac{2x}{-6} \quad \frac{x}{-3} \quad \frac{2x}{3}$

$= \boxed{\frac{2}{(2x+3)(x-3)}, x \neq 3, -\frac{3}{2}}$

11) $\frac{2x^{-1}+(x-1)^{-1}}{(2x)^{-2}}$

$\dfrac{\frac{2}{x} + \frac{1}{x-1}}{\frac{1}{4x^2}}$ CD: $4x^2(x-1)$

$\frac{8x(x-1)+4x^2}{x-1} = \frac{8x^2-8x+4x^2}{x-1}$

$\frac{12x^2-8x}{x-1} = \boxed{\frac{4x(3x-2)}{x-1}, x \neq 1, 0}$

12) $\frac{2}{7-x} + \frac{5x}{2x+3} - \frac{6}{2x^2-11x-21}$

$-x+7$

$-(x-7)$

$(2x+3)(x-7)$

$\frac{-2}{x-7} + \frac{5x}{2x+3} - \frac{6}{(2x+3)(x-7)}$

$\frac{-2(2x+3)}{(x-7)(2x+3)} + \frac{5x(x-7)}{(x-7)(2x+3)} - \frac{6}{(x-7)(2x+3)}$

$\frac{-2(2x+3)+5x(x-7)-6}{(x-7)(2x+3)}$

$\frac{-4x-6+5x^2-35x-6}{(x-7)(2x+3)} = \boxed{\frac{5x^2-39x-12}{(x-7)(2x+3)}, x \neq 7, -\frac{3}{2}}$

Solve the following rational equations. multiply both sides of the equation by the CD

13) $\left[\frac{x+5}{x-2} = \frac{5}{x+2} + \frac{28}{x^2-4}\right](x-2)(x+2)$

$(x+5)(x+2) = 5(x-2) + 28$

$x^2+7x+10 = 5x-10+28$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x = -4, 2 \quad x \neq \pm 2$

✓ x

$\boxed{x=-4}$

14) $\left[\frac{1}{6x^2} = \frac{1}{3x^2} - \frac{1}{x}\right] 6x^2$

$1 = 2 - 6x$

$\frac{-1}{-6} = \frac{-6x}{-6}$

$\boxed{\frac{1}{6} = x} \quad x \neq 0$

15) $\left[\frac{6}{x} - \frac{2}{x+3} = \frac{3(x+5)}{\underset{x(x+3)}{x^2+3x}}\right] x(x+3)$

$6(x+3) - 2x = 3(x+5)$

$6x+18-2x = 3x+15$

$4x+18 = 3x+15$

$x = -3 \quad x \neq 0, -3$

x

$\boxed{\text{No Solution}}$

16) $\left[\frac{7}{2x+1} - \frac{8x}{2x-1} = -4\right](2x+1)(2x-1)$

$7(2x-1) - 8x(2x+1) = -4(2x+1)(2x-1)$

$14x-7-16x^2-8x = -4(4x^2-2x+2x-1)$

$-16x^2+6x-7 = -16x^2+4$

$+16x^2 \qquad +16x^2$

$6x-7=4$

$6x=11$

$\boxed{x = \frac{11}{6}, \; x \neq \pm\frac{1}{2}}$

17) $\frac{2}{x+3} - \frac{3}{\underset{-(x-4)}{4-x}} = \frac{2x-2}{\underset{(x+3)(x-4)}{x^2-x-12}}$

$\left[\frac{2}{x+3} + \frac{3}{x-4} = \frac{2x-2}{(x+3)(x-4)}\right](x+3)(x-4)$

$2(x-4) + 3(x+3) = 2x-2$

$2x-8+3x+9 = 2x-2$

$5x+1 = 2x-2$

$3x = -3$

$\boxed{x=-1 \;, x \neq 4, -3}$

18) $\left[\frac{3}{x-2} + \frac{7x+1}{\underset{(x-2)(x+2)}{x^2-4}} = \frac{2}{x+2}\right](x-2)(x+2)$

$3(x+2) + 7x+1 = 2(x-2)$

$3x+6+7x+1 = 2x-4$

$10x+7 = 2x-4$

$8x = -11$

$\boxed{x = \frac{-11}{8} \quad x \neq \pm 2}$

19) Elizabeth and Nathan can paint their office in 5 hours working together. Being a professional painter, Elizabeth can paint twice as fast as Nathan. How long would it take Nathan to paint the office by himself?

$E + N = T$

$\left[\frac{1}{x} + \frac{1}{2x} = \frac{1}{5}\right]10x \rightarrow$

$10 + 5 = 2x$

$15 = 2x$

$7.5 = x$

It would take Nathan 15 hours to paint the office by himself.

20) Jennifer can clean a house twice as fast as Katelyn can. Working together, they can clean the house 3 hours faster than Jennifer can working alone. How long would it take Katelyn to clean the house?

$J + K = T$

$\left[\frac{1}{x} + \frac{1}{2x} = \frac{1}{x-3}\right]2x(x-3) \rightarrow$

$2(x-3) + (x-3) = 2x$

$2x - 6 + x - 3 = 2x$

$3x - 9 = 2x$

$x = 9$

Katelyn alone takes 18 hours to clean the house.

21) Bo and Duke can work together and paint a house in 3 hours. Working alone, it takes Bo two hours longer than Duke. How long does it take each to paint the house?

$B + D = T$

$\left[\frac{1}{x+2} + \frac{1}{x} = \frac{1}{3}\right]3x(x+2) \rightarrow$

$3x + 3(x+2) = x(x+2)$

$3x + 3x + 6 = x^2 + 2x$

$6x + 6 = x^2 + 2x$

$0 = x^2 - 4x - 6$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-6)}}{2(1)}$

$x = \frac{4 \pm \sqrt{40}}{2}$

$x = 5.16, \cancel{-1.16}$

Bo takes 7.16 hours and Duke takes 5.16 hours

22) Roxanne can clean twice as fast as Billy Jean. Working together, they can clean a house 3 hours faster than Roxanne can working alone. How long would it take Billy Jean to clean the house alone?

*Same as #20

$R + BJ = T$

$\left[\frac{1}{x} + \frac{1}{2x} = \frac{1}{x-3}\right.$

Find the intervals over which the following functions are increasing and over which the functions are decreasing. Identify the critical values and identify them as absolute or relative.

23) $f(x) = 5x^4 + 8x^3 - 6x^2 - 8x + 2$

Rel. Min. : $(-1.424, -1.316)$
Abs. Min : $(0.654, -2.646)$
Rel. Max : $(-0.43, 3.865)$
Increasing : $(-1.424, -0.43) \cup (0.654, \infty)$
Decreasing : $(-\infty, -1.424) \cup (-0.43, 0.654)$

24) $f(x) = -3x^3 + 2x^2 + 5x - 1$

Rel. Min. : $(-0.56, -2.65)$
Rel. Max. : $(1, 3)$
Increasing : $(-0.56, 1)$
Decreasing : $(-\infty, -0.56) \cup (1, \infty)$

25) $f(x) = 2x^3 - 3x^2 - 12x$

Rel. Max. : $(-1, 7)$
Rel. Min. : $(2, -20)$
Increasing : $(-\infty, -1) \cup (2, \infty)$
Decreasing : $(-1, 2)$

Find the asymptotes and intercepts of the functions below, and then graph the function.

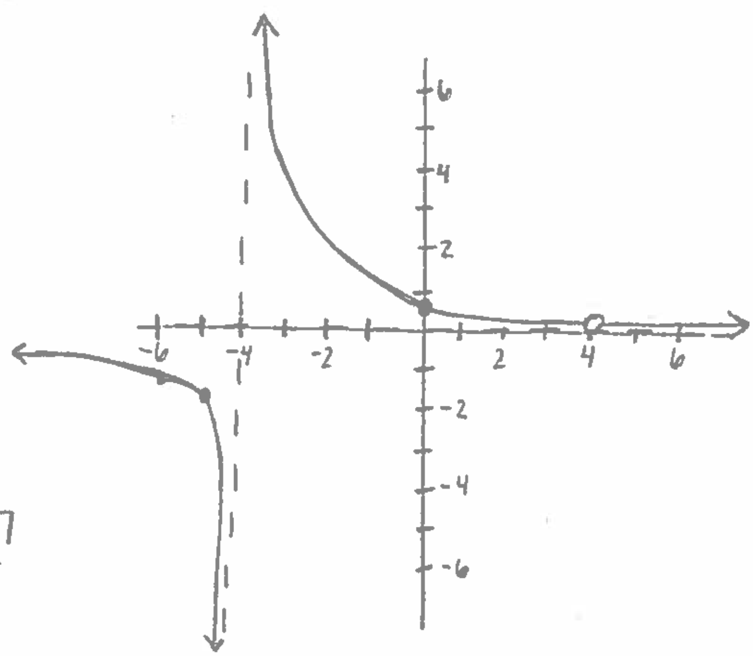
26) $f(x) = \dfrac{3x - 12}{x^2 - 16}$ D:1 D:2

$f(x) = \dfrac{3\cancel{(x-4)}}{\cancel{(x-4)}(x+4)}$

$f(x) = \dfrac{3}{x+4}$

HA: $y = 0$
SA: none
Hole: $(4, \frac{3}{8})$
VA: $x = -4$
x-int: none
y-int: $(0, \frac{3}{4})$

x	y
-5	-1.67
-6	-1.5



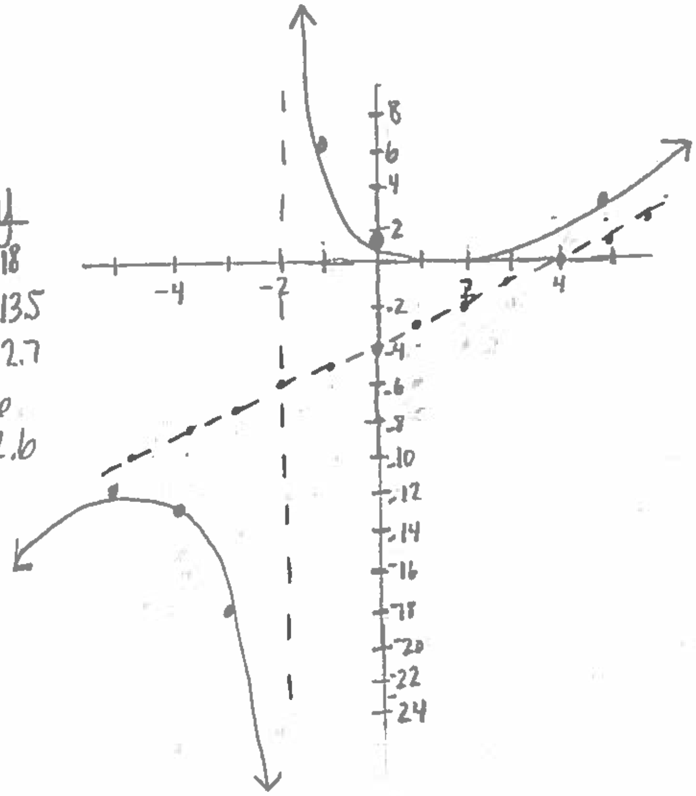
27) $f(x)=\dfrac{x^2-2x+3}{x+2}$ D:2 D:1

$f(x)= \dfrac{x^2-2x+3}{(x+2)}$

HA: none
SA: $y=x-4$
Hole: none
VA: $x=-2$
x-int: none
y-int: $(0, 3/2)$

$$\begin{array}{r|l} & x-4 \\ \hline x+2 & x^2-2x+3 \\ & -(x^2+2x) \\ \hline & -4x+3 \\ & -(-4x-8) \\ \hline & 11 \end{array}$$

x	y
-3	-18
-4	-13.5
-5	-12.7
-1	6
5	2.6

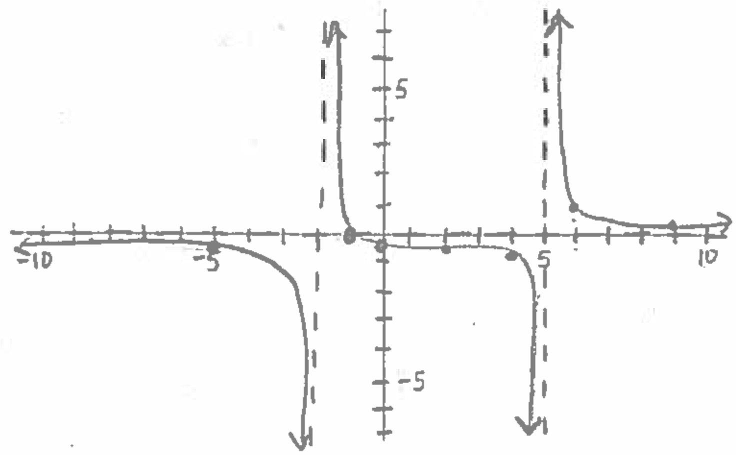


28) $f(x)=\dfrac{x+1}{x^2-3x-10}$ D:1 D:2

$f(x)= \dfrac{x+1}{(x-5)(x+2)}$

HA: $y=0$
SA: none
Hole: none
VA: $x=5, x=-2$
x-int: $(-1,0)$
y-int: $(0,-\frac{1}{10})$

x	y
2	-.25
-5	-.13
6	.88
9	.23

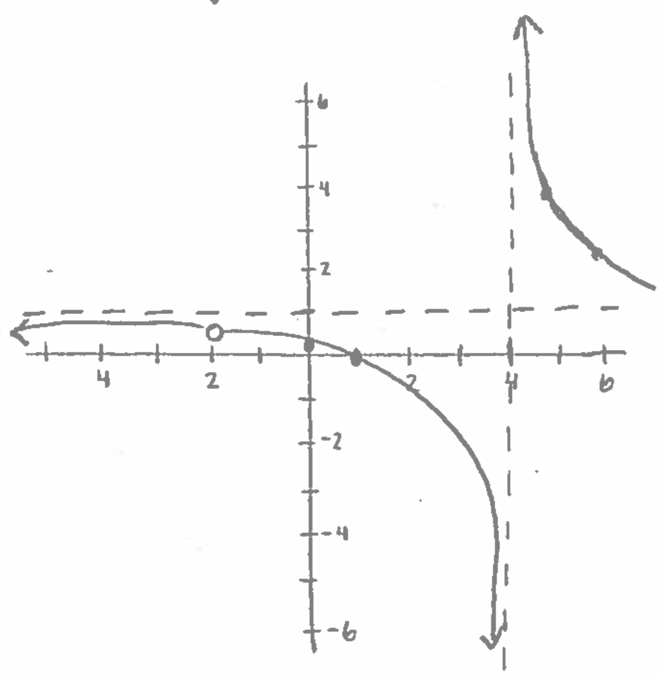


29) $f(x)=\dfrac{x^2+x-2}{x^2-2x-8} = \dfrac{(x+2)(x-1)}{(x-4)(x+2)}$

$f(x)= \dfrac{x-1}{x-4}$

HA: $y=1$
SA: none
Hole: $(-2,\frac{1}{2})$
VA: $x=4$
x-int: $(1,0)$
y-int: $(0, 1/4)$

x	y
5	4
6	2.5



30) Find a rational function that has the given condition.

The zero is at x = -1, the y-intercept is (0, 2), the asymptotes are at x = -2, x = 3 and y = 0.
There is a removable discontinuity (hole) at x = 4.

VA

HA

(higher degree in denominator)

$$\boxed{y = \frac{-12(x-4)(x+1)}{(x+2)(x-3)(x-4)}}$$

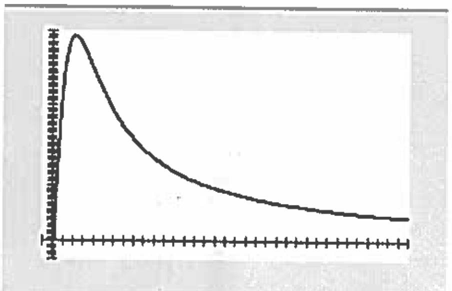
y-int: $\frac{-4}{24} = -\frac{1}{6}x = 2$

$x = -12$

31) The population P, in thousands, of a senior community is given by $P(t) = \frac{500t}{2t^2+9}$ where t is the time, in months.

a) Use your calculator to graph the function.

(You may need to adjust your window to see the maximum and what x is approaching for c) and e) below. It does not have to be exactly what I have below. Just be sure you can tell where the maximum is and what the function is approaching as t approaches positive infinity.



```
WINDOW
 Xmin=-1
 Xmax=35
 Xscl=1
 Ymin=-5
 Ymax=60
 Yscl=1
 Xres=1
```

b) Find the population at *t = 0, 1, 3, and 8 months.*

$P(t) = \frac{500t}{2t^2+9}$

$P(0) = 0$ 0 months → population = 0 senior citizens

$P(1) = 45.4545$ 1 month → population = 45.45 thousand senior citizens

$P(3) = 55.5556$ 3 months → population = 55.55 thousand senior citizens

$P(8) = 29.19708$ 8 months → population = 29.197 thousand senior citizens

c) Find the horizontal asymptote of the graph and complete the following: $P(t) \rightarrow$ <u>0</u> as $t \rightarrow \infty$

HA: P=0

d) Explain the meaning of the answer to part (c) in terms of the application.

As time increases, the population of senior citizens of this senior community decreases.

e) Find the maximum population and the value of *t* that will yield it.

(2nd Calc – Max)

Max: (2.12, 58.93)

The population of this senior community will reach its maximum population of 58.93 thousand senior citizens at 2.12 months.