Name: _____ Date: _____ Block: _____

Q1T2: Study Guide — Honors PreCalculus

ALL WORK IS TO BE DONE ON YOUR OWN PAPER

Remember that this is a study GUIDE and not the only material you should study. Studying only the problems that appear on this review guide will not be sufficient. You should also review problems from class starters, notes, and homework assignments for additional practice.

Graph the following functions. Identify the domain, range, and asymptotes.

1. $f(x) = -3\left(\frac{1}{2}\right)^{x-1}$

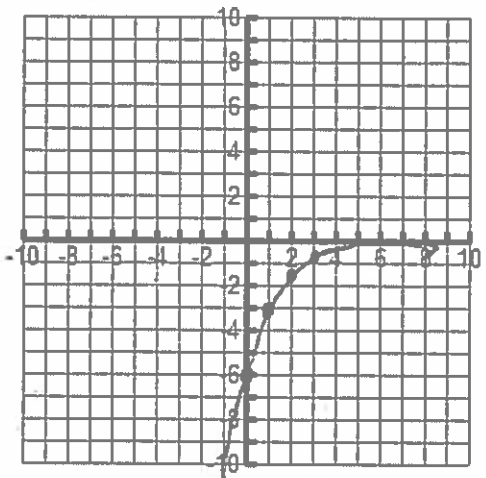
reflect over x-axis (-y)
vertical stretch of 3 (3y)
right 1 (x+1)

$y = \frac{1}{2}^x$ $x+1$ $-3y$

x	y
-2	4
-1	2
0	1
1	1/2
2	1/4

x	y
-1	-1/2
0	-6
1	-3
2	-3/2
3	-3/4

D: $(-\infty, \infty)$
R: $(-\infty, 0)$
HA: $y = 0$



2. $f(x) = \frac{1}{2}(4)^x + 1$

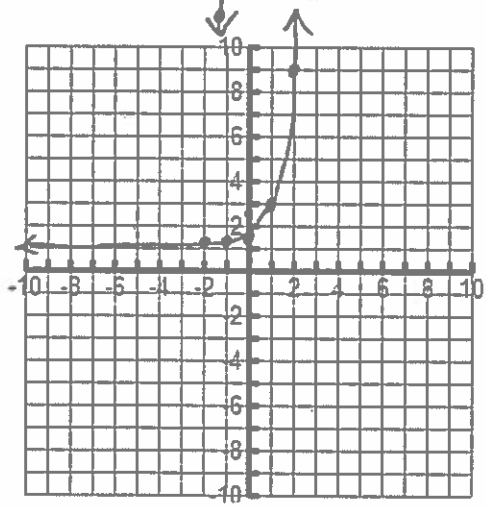
vertical shrink of 1/2 (1/2y)
up 1 (y+1)

$y = 4^x$ $\frac{1}{2}y+1$

x	y
-2	1/16
-1	1/4
0	1
1	4
2	16

x	y
-2	33/32 = 1.03
-1	9/8 = 1.125
0	1.5
1	3
2	9

D: $(-\infty, \infty)$
R: $(1, \infty)$
HA: $y = 1$



3. $f(x) = \log_3(x-2)$

right 2 (x+2)

$y = \log_3 x$
$x = \log_3 y$
$y = 3^x$

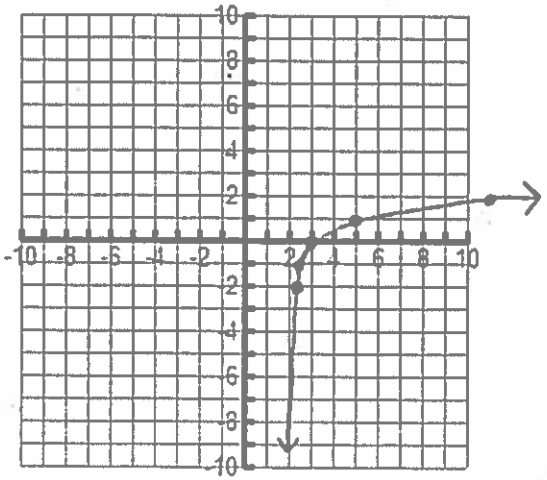
$x+2$

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

x	y
1/9	-2
1/3	-1
1	0
3	1
9	2

x	y
2 1/9	-2
2 1/3	-1
3	0
5	1
11	2

D: $(2, \infty)$
R: $(-\infty, \infty)$
VA: $x = 2$



4. $f(x) = 2\left(\frac{1}{3}\right)^{x-1}$

vertical stretch of 2 (2y)

right 1 (x+1)

$y = \frac{1}{3}^x$

x	y
-2	9
-1	3
0	1
1	1/3
2	1/9

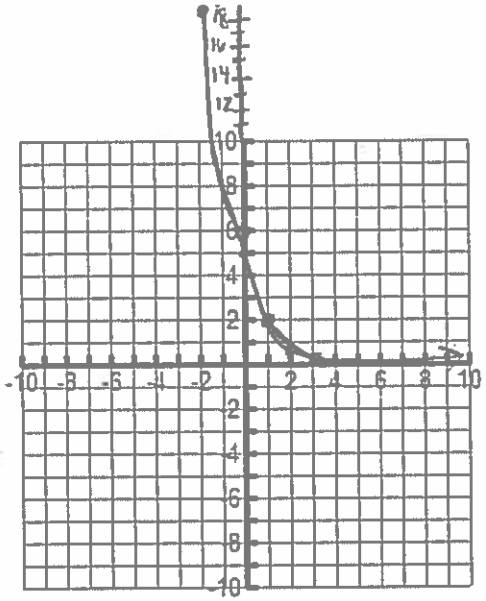
x+1 2y

x	y
-1	18
0	6
1	2
2	2/3
3	2/9

D: $(-\infty, \infty)$

R: $(0, \infty)$

HA: $y = 0$



5. $f(x) = -2^{x+1} - 3$

reflect over x-axis (-y)

left 1 (x-1)

down 3 (y-3)

$y = 2^x$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

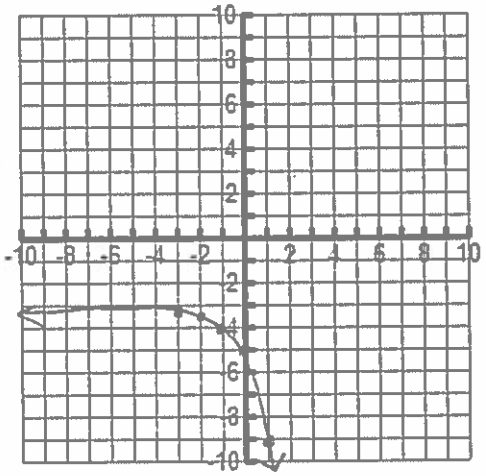
x-1 -y-3

x	y
-3	-3 1/4
-2	-3 1/2
-1	-4
0	-5
1	-7

D: $(-\infty, \infty)$

R: $(-\infty, -3)$

HA: $y = -3$



6. $f(x) = \log_2(x + 3) - 1$

left 3 (x-3)

down 1 (y-1)

$y = \log_2 x$

$x = \log_2 y$

$y = 2^x$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2

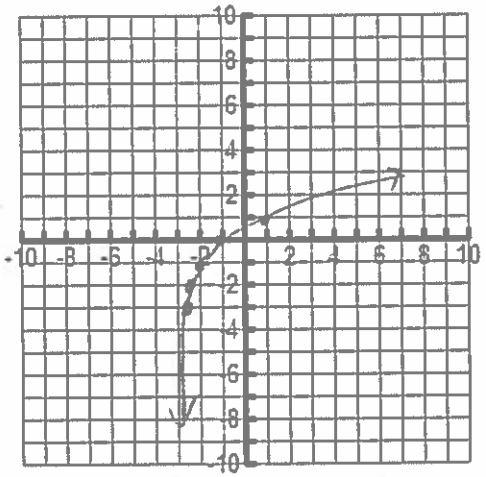
x-3 y-1

x	y
-2 3/4	-3
-2 1/2	-2
-2	-1
-1	0
1	1

D: $(-3, \infty)$

R: $(-\infty, \infty)$

VA: $x = -3$



7. Find the equation of the exponential function that passes through the points (1,6) and (0,2).

$y = ab^x$

$2 = ab^0$ $\quad 6 = ab^1$

$2 = a$ $\quad 6 = ab$

$6 = 2b$

$3 = b$

$\boxed{y = 2(3)^x}$

8. Find the equation of the exponential function that passes through the points (2,0.75) and (-3,24).

$y = ab^x$

$.75 = ab^2$ $\quad 24 = ab^{-3}$

$\frac{.75}{b^2} = a$ $\quad 24 = \frac{.75}{b^2} \cdot b^{-3} \rightarrow 24 = \frac{.75}{b^5} \cdot b^5$

$24b^5 = .75$

$b^5 = \frac{1}{32}$

$b = \frac{1}{2}$

$\frac{.75}{b^2} = a$

$\frac{.75}{(\frac{1}{2})^2} = a$

$3 = a$

$\boxed{y = 3(\frac{1}{2})^x}$

Compound Interest and Continuously Compounded Interest Formulas

- *Compound Interest Formula:* $A = P\left(1 + \frac{r}{n}\right)^{nt}$
- *Continuously Compounded Interest Formula:* $A = Pe^{rt}$

9. If $3,500 is invested in an account that pays 4.5% interest compounded quarterly, how much money is in the account at the end of 20 years? How much more would be in the account if it were compounded continuously?

$P = 3500$

$r = .045$

$n = 4$

$A = 3500\left(1 + \frac{.045}{4}\right)^{4t}$

$t = 20 \rightarrow A = 3500(1.01125)^{4(20)}$

$\boxed{A = \$8565.46 \text{ quarterly}}$

$P = 3500$

$r = .045$

$A = 3500e^{.045t}$

$t = 20 \rightarrow A = 3500e^{.045(20)}$

$\boxed{A = \$8608.61 \text{ continuously}}$

$\boxed{\$43.15 \text{ more}}$

10. If $4,750 is invested in an account that has an annual interest rate of 3.75% compounded semiannually, how long will it take for the investment to earn $3,000 in interest?

$P = 4750$

$r = .0375$

$n = 2$

$A = 4750\left(1 + \frac{.0375}{2}\right)^{2t}$

3000 in interest means there is $3000 + $4750 = $7750 in the account

$7750 = 4750\left(1 + \frac{.0375}{2}\right)^{2t}$

$1.6316 = 1.01875^{2t}$

$\log_{1.01875} 1.6316 = 2t$

$26.3539 = 2t$

$13.177 = t$

It would take around 13.177 years to earn $3000 in interest.

11. How long would it take for an investment to double in an account that has an interest rate of 4.25% compounded continuously?

$r = .0425$

$A = Pe^{rt}$

$\frac{2P}{P} = \frac{Pe^{.0425t}}{P}$

$2 = e^{.0425t}$

$\ln 2 = .0425t$

$.693 = .0425t \rightarrow 16.31 = t$

It would take around 16.31 years for the investment to double.

12. How much money must be invested in account with an annual interest rate of 3% compounded monthly so that there is $50,000 in the account 10 years after the initial investment?

$r = .03$

$n = 12$

$A = 50000$

$t = 10$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$50000 = P\left(1 + \frac{.03}{12}\right)^{12(10)}$

$50000 = P(1.0025)^{120}$

$\frac{50000}{1.349} = \frac{P(1.349)}{1.349}$

$\$37054.78 = P$

$37054.78 should be invested in the account

Logarithmic Expressions

Evaluate the following logarithmic expressions. Round to three decimal places if necessary.

13. $\log_4 \frac{1}{64} = X$

$4^X = \frac{1}{64}$

$\boxed{X = -3}$

14. $\log_{81} \frac{1}{9} = X$

$81^X = \frac{1}{9}$

$\boxed{X = -\frac{1}{2}}$

15. $\log_6 17 = X$

$6^X = 17$

$\frac{\log 17}{\log 6} = \boxed{1.581}$

16. $\log_3 81 = X$

$3^X = 81$

$\boxed{X = 4}$

17. $\log_2 0.125 = X$

$\log_2 \frac{1}{8} = X$

$2^X = \frac{1}{8}$

$\boxed{X = -3}$

18. $\ln e^5 = X$

$e^X = e^5$

$\boxed{X = 5}$

19. $7^{2\log_7 6} = X$

$\log_7 X = \log_7 6^2$

$\log_7 X = \log_7 36$

$\boxed{X = 36}$

20. $\log_5 15 = X$

$5^X = 15$

$\frac{\log 15}{\log 5} = \boxed{1.683}$

21. $\log_4 \sqrt{2} = X$

$4^X = \sqrt{2}$

$(2^2)^X = 2^{1/2}$

$2^{2X} = 2^{1/2}$

$2X = 1/2$

$\boxed{X = 1/4}$

22. $\log_2 16 + \log_3 \frac{1}{9}$

$2^X = 16 \quad 3^X = \frac{1}{9}$

$X = 4 \quad X = -2$

$4 + (-2)$

$\boxed{2}$

23. $10^{\log 4 = X} - \log_5 125 + \log_6 \frac{1}{216}$

$\log X = \log 4 \quad 5^X = 125 \quad 6^X = \frac{1}{216}$

$X = 4 \quad X = 3 \quad X = -3$

$4 - 3 + (-3)$

$\boxed{-2}$

24. Evaluate the following expression: $(\log_5 6)(\log_6 25)$

$\frac{\log 6}{\log 5} \cdot \frac{\log 25}{\log 6} = \frac{\log 25}{\log 5} = \log_5 25 = x$

$5^x = 25$

$\boxed{x=2}$

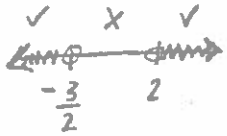
25. Find the domain of the following logarithmic expression: $f(x) = \log_4(2x^2 - x - 6)$

$2x^2 - x - 6 > 0$

$(2x+3)(x-2) > 0$

Test the intervals

$x > -\frac{3}{2}$ $x > 2$



$\boxed{D: (-\infty, -\frac{3}{2}) \cup (2, \infty)}$

Properties and Laws of Logarithms

Rewrite the following expressions in a form with no logarithm of a product, quotient, root, or power. In other words, rewrite the expression in expanded form.

26. $\log_3 \frac{x^4\sqrt{y}}{9z^3}$ $4\log_3 x + \frac{1}{2}\log_3 y - \log_3 9 - 3\log_3 z$

$3^x = 9$, $x = 3$

$\boxed{4\log_3 x + \frac{1}{2}\log_3 y - 3 - 3\log_3 z}$

27. $\ln\left(\frac{x+3}{\sqrt[3]{x^2-4}}\right)^2$ $2\ln(x+3) - \frac{2}{3}\ln(x-2) - \frac{2}{3}\ln(x+2)$

$(x-2)(x+2)$

28. $\log \frac{10^y\sqrt{2y-1}}{3y+1}$ $y \log 10 + \frac{1}{2}\log(2y-1) - \log(3y+1)$

$10^x = 10$, $x = 1$

$\boxed{y + \frac{1}{2}\log(2y-1) - \log(3y+1)}$

Rewrite the expression as a single logarithm. You do not have to rationalize.

29. $5\log_4(x-1) + \log_4 x^2 - 2\log_4(2x+3)$

$\log_4 \frac{x^2(x-1)^5}{(2x+3)^2}$

30. $3[\ln x + \ln(2x+1)] - 4\ln(x-3)$

$\ln \frac{x^3(2x+1)^3}{(x-3)^4}$

31. $\frac{1}{3}\log_2(2x+1) - \frac{1}{2}[\log_2(x+5) + \log_2 x] + 2\log_2 x$

$\log_2 \frac{x^2\sqrt[3]{2x+1}}{\sqrt{x(x+5)}}$ or $\log_2 \frac{x^2\sqrt[3]{2x+1}}{\sqrt{x^2+5x}}$

Exponential and Logarithmic Equations

Solve the following equations. If necessary, round your answer to three decimal places.

32. $10^{2x-1} = 5$

$\log 5 = 2x - 1$

$.69897 = 2x - 1$

$1.69897 = 2x$

$\boxed{.849 = x}$

33. $5 - 2^{3x+1} = 3$

$-2^{3x+1} = -2$

$2^{3x+1} = 2^1$

set exponents equal since you have the same base

$3x + 1 = 1$

$3x = 0$

$\boxed{x = 0}$

34. $3e^{x+1} - 1 = 2$

$3e^{x+1} = 3$

$e^{x+1} = 1$

$\ln 1 = x + 1$

$0 = x + 1$

$\boxed{-1 = x}$

35. $\log_5(x - 1) = 4$

$5^4 = x - 1$

$625 = x - 1$

$\boxed{626 = x}$

36. $\log_3(x - 1) = 1 + \log_3(x + 2)$

$\log_3(x-1) - \log_3(x+2) = 1$

$\log_3\left(\frac{x-1}{x+2}\right) = 1$

$(x+2)3^1 = \frac{x-1}{x+2} \cdot x+2$

$3x + 6 = x - 1$

$2x = -7$

$x \neq -\frac{7}{2}$ **No Solution**

37. $\ln(2x + 1) = 3 + \ln(x + 1)$

$\ln(2x+1) - \ln(x+1) = 3$

$\ln \frac{2x+1}{x+1} = 3$

$(x+1)e^3 = \frac{2x+1}{x+1} \cdot x+1$

$20.086x + 20.086 = 2x + 1$

$-2x \quad -2x$

$18.086x + 20.086 = 1$

$18.086x = -19.086$

$x \neq -1.055$

No Solution

38. $\log_2(x+2) - \log_2 x + \log_2(2x-1) = 3$

$\log_2 \frac{(x+2)(2x-1)}{x} = 3$

$2^3 = \frac{2x^2+3x-2}{x}$

$x \cdot 8 = \frac{2x^2+3x-2}{x} \cdot x$

$8x = 2x^2+3x-2$

$0 = 2x^2-5x-2$

$x = \frac{-(-5) \pm \sqrt{(-5)^2-4(2)(-2)}}{2(2)} = \frac{5 \pm \sqrt{41}}{4} = x$

$x = \frac{5+\sqrt{41}}{4} = \boxed{2.851}$

$x \neq \frac{5-\sqrt{41}}{4} \neq -.351$

39. $e^{2x} + 8e^x - 9 = 0$

$\frac{9}{9}x\frac{-1}{-1} = -9 \quad \frac{e^x}{9} \quad \frac{e^x}{-1}$

$\frac{9}{9}+\frac{-1}{-1} = 8$

$(e^x+9)(e^x-1) = 0$

$e^x+9=0 \quad e^x-1=0$

$e^x = -9 \quad e^x = 1$

$\ln(-9) = x \quad \ln 1 = x$

No Solution $\quad \boxed{x=0}$

40. $4x^2(10^x) - 11x(10^x) + 6(10^x) = 0$

$10^x(4x^2-11x+6) = 0$

$10^x = 0 \quad 4x^2-11x+6=0$

no solution $\quad (x-2)(4x-3)=0$

$\boxed{x = 2, 3/4}$